Texto para Discussão

Série Economia

**OBSERVABILITY AND ENDOGENOUS
ORGANIZATIONS**

Prof. Dr. Gabriel de Abreu Madeira

Universidade de São Paulo

Faculdade de Economia, Administração e Contabilidade
de Ribeirão Preto

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Observability and Endogenous Organizations

Weerachart T. Kilenthong[*] and Gabriel A. Madeira[†]

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Abstract

This paper establishes a relationship between the observability of common shocks and optimal organizational design under a multiagent moral hazard environment. We show that, with sufficient information about common shocks, a cooperative organization can be optimal even if outputs are highly correlated. This is consistent with the empirical observation that cooperative arrangements may be more prevalent when outputs are correlated. The model is then embedded in a Walrasian equilibrium model where choices of organization and investment on information about common shocks are determined jointly. Numerical results reveal that both cooperative and individualistic regimes can coexist in equilibrium. The interplay between organization, investment in information, and inequality is thus analyzed.

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1 Introduction

Economic agents are often organized in cooperative groups, where they take decisions jointly and share relevant information. These cooperative arrangements, however, coexist with more individualistic, competitive regimes. Several examples show that even members of apparently similar communities, act coordinately in some cases and independently in others. Data from Indian villages (e.g., Townsend and Mueller, 1998; Mueller et al., 2002) show that while some landowners have their land farmed by groups of individuals acting cooperatively, others divide up their land in several plots, and employ a single worker or family in each of them. This duality between cooperative and individualistic arrangements has also been observed in credit contracts: many microcredit

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[*]University of the Thai Chamber of Commerce, Bangkok, Thailand, Email: weerachart_kil@utcc.ac.th

[†]Universidade de Sao Paulo, Email: gmadeira@uchicago.edu

arrangements are one-per-credit contracts, but recently group joint liability credit has been widely adopted in developing economies (Ahlin and Townsend (2007)). Such variability can also be observed in developed economies: big industrial conglomerates many times coexists with smaller firms producing similar goods. In this paper, we present a model that suggests that the availability of information about aggregate shocks may be an important ingredient to determine if individuals will be organized one way or the other.

This variability of organizational regimes has motivated a theoretical literature, based on multi-agent moral hazard models (e.g., Holmstrom and Milgrom, 1990; Ramakrishnan and Thakor, 1991; Itoh, 1993; Prescott and Townsend, 2002), which establishes that cooperation or competition can be optimal, depending on the economic environment. The contractual advantage of cooperation comes from the fact that, when workers act cooperatively, there are incentives for mutual enforcement of optimal levels of effort. Cooperative groups, however, can potentially collude and make side payments to mitigate incentives. On the other hand, the correlation of outputs is informative about the efforts of others. Hence, relative performance will be an optimal arrangement when outputs are strongly correlated across agents. This result conflicts with the empirical observation that cooperative arrangements are sometimes more prevalent when outputs are highly correlated (e.g., Ahlin and Townsend, 2007).

Our model overcomes this inconsistency by adding the possibility of (costly) observability (Holmstrom, 1979) of common shocks that drives the correlation of the outputs. We prove that even with highly correlated outputs, perfect observability of common shocks will make cooperative arrangements optimal. Intuitively, from the contractual point of view, outputs are not correlated when the common shock is perfectly observed. Therefore, as in Ramakrishnan and Thakor (1991); Itoh (1993), the relative performance regime looses the advantage, and the group regime is dominating even though, from an ex ante perspective, outputs are highly correlated.

We also put the standard multiagent moral hazard model into a Walrasian equilibrium framework where individuals trade memberships in productive organizations (e.g., Prescott and Townsend, 2006). The organizational affiliation of each individual and the investment on observability of common shocks are jointly determined in a competitive equilibrium model. When endogenous organization and investment on observability are jointly modeled, the relationship between organizational format and output correlation is subtler. If aggregate shocks are too relevant, there are strong incentives for investment on information. If such investment produces perfect observability of outputs, cooperation will certainly emerge. Also, the willingness to pay for information or the value of information should be heterogeneous across individuals with different wealths. Results about

these interplays between wealth distribution, organization and information acquisition, as provided by our equilibrium formulation, can help interpreting cross sectional data and generate testable implications for the model.

Our results may help explaining the transitions between cooperative and individualistic regimes that are often observed. Indeed, some studies (e.g. Barkey and Van Rossem (1997)) have documented instability of organization over times. Madeira and Townsend (2008) present a dynamic model of endogenous groups where the progression of utility promises over time drives changes in inequality and wealth that originate formation and dissolution of groups[1]. In steady state, both regimes coexists, with transitions in both directions. However, this stationary persistence of transitions contrasts with the waves of mass regime switching, such as the corporate mergers in the late 90's and the relatively recent proliferation of joint liability microcredit contracts. From our model, those can possibly result from advances in information technology.

This paper also belongs to the risk sharing literature. Individuals inside our cooperative groups share risk, so they can be interpreted as risk sharing groups or networks. Although some groups of individuals achieve successful amounts of risk sharing (Fafchamps and Gubert (2007), Goldstein (2000) and Grimard (1997)), most of the empirical tests reject risk sharing within villages (e.g. Deaton (1992), Townsend (1994) and Jalan and Ravallion (1999)). Further, Chiappori et al. (2006) shows that some family networks and villages in Thailand are successful to share risk, but others perform poorly.

The remaining of the paper is organized as follows. Section 2 describes the basic environment, the informational structure and the process of information acquisition. Section 3 presents the two possible productive organizational regimes, and characterizes optimal arrangements within them. It also presents theoretical and numerical results determining how optimality of organizational type relates with availability of common information. Section 4 models Walrasian markets of memberships in the organizations. The choices of investment in information and organizational affiliation are jointly determined by an equilibrium in this market. The welfare theorems and the existence theorem are proved in Section 5. Section 6 presents numerical examples of the competitive equilibria. Section 7 concludes the paper.

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[1]The relationship between wealth, inequality and optimal organizational regime is the object of Prescott and Townsend (2002)

2 The Model Economy

There are two physical commodities, a consumption good, c and a capital good, k. For simplicity, we assume that consumption can only take on finite (but possibly many) levels[2]. There is a continuum of risk averse agents which are divided into I types, each of which is indexed by $i = \{1, 2, \ldots, I\}$. Each type i consists of a $\alpha^i \in (0, 1)$ fraction of the population, and $\sum_i \alpha^i = 1$. Each agent type i is endowed with κ^i units of capital. The endowment is the main source of heterogeneity among agents.

There is a continuum of local economies where production is carried on by a fixed amount of workers. Those can be interpreted as small villages or firms. The productive technology in these local economies depend on a local underlying state of nature ω, which affects the output of all local workers. A state space Ω is the set of S states of nature, i.e., $\Omega = \{\omega_1, \omega_2, \ldots, \omega_S\}$, and $Pr(\omega)$ is the probability of state ω occurring, so $\sum_{\omega \in \Omega} Pr(\omega) = 1$. The underlying state of nature ω is not necessarily observed. But the elements of a partition of the state space Ω can be observed after the realization of output. This partition characterizes an informational regime, precisely defined in the next section.

An agent i derives utility from consumption, c^i, and effort, e^i, according to the utility function $U^i(c^i, e^i)$. The utility function is assumed to be concave and strictly increasing on c^i, and decreasing on e^i. Again, we assume that effort, or action, can take on finite (possibly many) values. Effort is private information, and therefore there is a moral hazard problem. In addition, we assume that effort is exerted before we could possibly learn out the underlying state ω. That is, effort can not be contingent on ω.

Output q can be produced using effort e and capital k as inputs. Again there are finite (possibly many) feasible levels of output. We assume that all agents have the same productive technology. The production technology is however random and is given by $f(q|e, k, \omega)$, which is the probability of having output q conditional on effort e, capital k, and state ω. This probability satisfies

$$\sum_q f(q|e, k, \omega) = 1, \ \forall e, k, \omega \tag{1}$$

Note that the outputs of agents working for the same firm or local economy, which experience the same common shock ω, are possibly correlated:

$$\Pr(q_j \mid q_i, e_j, e_i, k) = \frac{\sum_\omega \pi(\omega) f(q_j \mid e_j, k, \omega) f(q_i \mid e_i, k, \omega)}{\sum_\omega \pi(\omega) f(q_i \mid e_i, k, \omega)}. \tag{2}$$

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[2]We can apply the limiting arguments used in Prescott and Townsend (1984a) to establish the results if the commodity space is not finite.

Notice that this correlation is explicitly modeled a result of common shocks. Conditional on the common shocks, outputs are uncorrelated across agents. This clearly violates the condition in Holmstrom (1979) under which the information will be valuable. However, we will show that this information about the common shocks still be valuable for multi-agent organizations. It is valuable under a multi-agent organization because this potential correlation provides useful information for incentives, especially when relative performance evaluation is an ingredient of contracts.

2.1 Informational Regimes

An information regime is a partition of Ω, under the information technology z and is denoted by F_z. Let a subset $A_s \subset \Omega$ be a typical member of an information structure, i.e. $A_s \in F_z$. With abusive of notation, we will call A_s an observed state s. For notational convenience, let S_z denote the number of subsets of Ω in F_z, that is, $F_z = \{A_1, \ldots, A_{S_z}\}$. For example, the coarsest informational structure (with the least information about states of nature) is $F_0 = \{(\omega_1, \omega_2, \omega_3, \omega_4)\}$. Under this informational regime, we cannot distinguish any common state from each other. On the other hand, the finest information structure (we can distinguish all states of nature from one another) is $F_2 = \{(\omega_1), (\omega_2), (\omega_3), (\omega_4)\}$.

For a given regime F_z, we can define a modified production function for each subset $A_s \in F_z$ as (using Bayes' rule)

$$Pr(q|e, k, s) = \frac{\sum_{\omega \in A_s} f(q|e, k, \omega) Pr(\omega)}{Pr(A_s)} \tag{3}$$

where $Pr(A_s) = \sum_{\omega \in A_s} Pr(\omega)$ is the probability of having an common state in A_s. In addition, this production function satisfies the following probability condition

$$\sum_q Pr(q|e, k, s) = \sum_q \frac{\sum_{\omega \in A_s} f(q|e, k, \omega) Pr(\omega)}{Pr(A_s)} = \frac{\sum_{\omega \in A_s} Pr(\omega) \sum_q f(q|e, k, \omega)}{Pr(A_s)} = 1 \tag{4}$$

Information Cost

The informational regime z can be thought as the amount of information available about ingredients that are relevant for local production. It could include, for instance, information about weather conditions or sectoral economic indicators. This kind of information can be obtained at some cost, so we allow for possibility that the informational partition can be made finer with some investment on information. An informational structure F_z can be obtained at a cost of $C(z)$. The cost is increasing in the accuracy of the information; $C(z) \geq C(z')$ for $F_z \subset F_{z'}$. In words, the cost is larger when the information regime is finer. This cost is assumed to be per unit of firms participating in the regime. The information cost is paid ex ante in units of capital.

3   Firms as Optimal Contracts: A Principal-Agent Formulation

Each individual is employed by a firm or local economy. Since workers are risk averse, they potentially benefit from risk-sharing contracts provided by a risk neutral outsider, or principal [3]. Effort is unobserved, however. Hence, there is moral hazard. As a result, it is optimal to impose output dependency of consumption in order to provide incentives. This is also the reason why it may be desirable to invest in observability of states of nature.

We consider two classes of organization determining the feasible contractual arrangements; (i) *relative performance firms*, and (ii) *group firms*.

Within a *group firm*, economic decisions are taken jointly between the agents. There is mutual observability of effort and side payments among workers. This is sometimes advantageous, as it allows mutual monitoring across workers: one individual will be punished by a bad performance of the others, so she will make sure that the others make the desirable amount of effort. However, mutual observability of effort is not always optimal: when outputs are too strongly correlated (which, in the current environment, means that common shocks are very important), there may be collusion among agents. Intuitively, all workers inside a firm could collude and coordinate with low effort: low output for all workers would look like a bad common shock.

Relative performance evaluation then emerges as an useful incentive tool. In *relative performance firms*, there is no mutual observability of efforts. Workers are placed in separate environments so they cannot share information about efforts, and each agent is rewarded separately. If one agent performs poorly compared with the others, she will be punished, while if she performs well compared with the others she will be rewarded. These comparative compensation schemes are particularly useful when outputs are highly correlated so the output of one individual is informative about the distribution of outputs of others.

In addition to organizational regime, a firm is described by the informational regime z, a capital input k, and an incentive compatible distribution of consumption. Without loss of generality, we can focus only on efficient firms, and these can be described from its organizational regime, its informational regime, its capital input, and the expected utility of each agent.

This section characterizes efficient firms, taking as given the capital input k, the informational regime z, and expected utility levels of each agent. The determination of k, z and utility promises follow from the competitive equilibrium. For expositional convenience, we assume that there are only two agents inside each firms, but this can be easily extended to the case of N agents.

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[3]In the general equilibrium decentralization, the outsider will be modeled as an intermediary

3.1 Relative Performance Firms

An efficient relative performance contract maximizes an expected surplus conditional on incentive and technological constraints, and also on minimal expected utility levels (outside options) for the individuals. Formally, a relative performance contract $\pi(c, q, e, s)$ is the probability that each agents will receive consumption $c = (c_1, c_2)$ respectively, their realized outputs are $q = (q_1, q_2)$, their efforts are $e = (e_1, e_2)$, and the observed state is s (or A_s). We will in fact use $\pi(c, q, e, s)$ as the decision variable.

The timing of events is as follows. First, each agent is randomly assigned a level of effort according to $Pr(e)$. After effort is performed, some state of nature $\omega \in A_s$ is realized (the informational set A_s is revealed), and the outputs q are obtained with probabilities determined by $Pr(q|e, k, s)$. Then, each agent receives consumption which is randomly assigned according to $Pr(c|q, e, s)$, that is allowed to depend on the observed informational set A_s. So, the decisions that are implicit in contracts are $Pr(e)$ and $Pr(c|q, e, s)$, but these two objects can be obtained from the joint distribution $\pi(c, q, e, s) = Pr(e)Pr(c|q, e, s)Pr(q|e, k, s)Pr(s)$.

As a probability measure, a lottery $\pi(c, q, e, s)$ must satisfy the following *probability constraint*:

$$\sum_{c,q,e,s} \pi(c, q, e, s) = 1, \quad \text{and } \pi(c, q, e, s) \geq 0, \ \forall (c, q, e, s) \tag{5}$$

A feasible contract $\pi(c, q, e, s)$ must also be consistent with the production technology (3). This condition is guaranteed using the following *mother nature constraint*. For any observed state $s \in F_z$, output levels $\overline{q}$, and for any $\overline{e}$ such that $\sum_{c,q,A_\omega} \pi(c, q, \overline{e}, s) > 0$:

$$\frac{\sum_c \pi(c, \overline{q}, \overline{e}, s)}{\sum_{c,q} \pi(c, q, \overline{e}, s)} = Pr(\overline{q}|\overline{e}, k, s) \tag{6}$$

where the joint production function is given by

$$Pr(\overline{q}|\overline{e}, k, s) = f(q_1|\overline{e}_1, k, s)f(q_2|\overline{e}_2, k, s) \tag{7}$$

The mother nature constraints can be rewritten as linear constraints:

$$\sum_c \pi(c, \overline{q}, \overline{e}, s) = \sum_{c,q} \pi(c, q, \overline{e}, s)Pr(\overline{q}|\overline{e}, k, s) \ \forall(\overline{q}, \overline{e}). \tag{8}$$

Since efforts are privately observed, contracts must be incentive compatible: each agent must have incentives to perform the recommended level of effort. The incentive constraints can be formulated as:

$$\sum_{c,q,e_{-i},s} \pi(c,q,\overline{e}_i,e_{-i},s)u_i(c_i,\overline{e}_i) \geq \sum_{c,q,e_{-i},s} \pi(c,q,\overline{e}_i,e_{-i},s)\frac{Pr(q|\widetilde{e}_i,e_{-i},k,s)}{Pr(q|\overline{e}_i,e_{-i},k,s)}u_i(c_i,\widetilde{e}_i), \ \forall \widetilde{e}_i \qquad (9)$$

The contract also needs to assure that individuals will get an utility level not smaller than the promised expected utility, $\overline{u}_i$. This participation constraint can be expressed as:

$$\sum_{c,q,e,s} \pi(c,q,e,s)u_i(c_i,e_i) \geq \overline{u}_i, \ \forall i \qquad (10)$$

Since effort is performed before the observed state s is revealed, the distribution of effort must be independent of the distribution of s. That is, for any vector of effort levels $\overline{e}$:

$$\sum_{c,q} \pi(c,q,\overline{e},s) = Pr(A_s)\sum_{c,q,s} \pi(c,q,\overline{e},\overline{s}), \quad \forall \overline{e} \ and \ \overline{s} \in \{1,...,S_z\} \qquad (11)$$

Constraints (5), (8) and (11) guarantee that the choosing $\pi(c,q,e,s)$ is equivalent to choosing $Pr(e)$ and $Pr(c|q,e,s)^4$.

An optimal relative performance policy $\pi^*(c,q,e,s)$ solves:

Program 1:

$$\max_{\pi} \sum_{c,q,e,s} \pi(c,q,e,s)(q_1 + q_2 - c_1 - c_2) \qquad (12)$$

subject to constraints (5)-(11).

3.2 Group Firms

Another organizational form available for the individuals in each location is the group regime, where decisions are taken jointly by both agents. Again, for a group, a contract can be characterized as $Pr(e)$ and $Pr(c|q,e,s)$. However given these objects, individuals inside a group can potentially collude and use a combination of coordinated effort levels and side payments to promote local Pareto improvements. The principal anticipate this possibility and focuses only on collusion-proof contracts. In these collusion proof contracts, whenever vectors of effort, e, and consumption, c, are recommended, it must be the case that, for a pair of given positive weights, μ_1 and μ_2:

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[4] Notice also that From (5) and (11), we also know that $\sum(c,q,e,\overline{s}) = Pr(A_{\overline{s}})$ for all $\overline{s} \in \{1,...,S_z\}$, that is, there is consistency between the probabilities in π and the probabilities of the informational sets given z.

$$\sum_{c,q,s} Pr(c|e,q,s)Pr(q|e,k,s) \sum_i \mu_i u_i(c_i) \geq$$

$$\sum_{c,q,s} Pr(c|\overline{e},q,s)Pr(q|\overline{e},k,s) \sum_i \mu_i u_i(\overline{c}_i(c)) \qquad (13)$$

for any $\overline{e}$ and any $\overline{c}(c) \equiv [\overline{c}_1(c), \overline{c}_2(c)]$ such that $\overline{c_1}(c) + \overline{c_2}(c) = c_1 + c_2 \equiv c_A$, where c_A can be interpreted as the common consumption within a group. When u_i is concave and separable $(u_i(c_i, e_i) = \widetilde{u}_i(c_i) - v_i)$, this implies that c is a function of c_A and $\mu \equiv (\mu_1, \mu_2)$ given by:

$$c(c_A, \mu) = \arg\max_c \sum_i \mu_i u_i(c_i) \qquad (14)$$

$$s.t. \sum_i c_i = c_A.$$

So, a group contract can be formulated as, first, the choice (possibly random) of the inside-group Pareto weights $(Pr(\mu))$, and then the choice (possibly random) of effort $(Pr(e|\mu))$ and finally of common consumption conditional on outputs, effort and μ $(Pr(c_A|q,e,s,\mu)$, (the choices of μ immediately determines the whole vector c as a function of c_A and μ). We will first characterize the constraints of the problem after the choice of $Pr(\mu)$. Again, the choices of $Pr(e|\mu)$ and $Pr(c_A|q,e,s,\mu)$ can now be expressed by the joint distribution (conditional on μ) $\pi_\mu(c_A, q, e, s) = Pr(e|\mu)Pr(q|e,k,s)Pr(c_A|q,e,s,\mu)Pr(A_s)$. Again, this joint distribution, π_μ, is a probability distribution, so:

$$\sum_{c_A,q,e,s} \pi_\mu(c_A, q, e, s) = 1, \ \ \pi_\mu(c_A, q, e, s) \geq 0, \ \ \forall(c_A, q, e, s). \qquad (15)$$

Again, π_μ must be subject to a *mother nature* constraint that guarantees consistency with the productive technology:

$$\sum_{c_A} \pi_\mu(c_A, \overline{q}, \overline{e}, \overline{s}) = Pr(q|\overline{e}, k, \overline{s}) \sum_{c_A,q} \pi_\mu(c_A, q, \overline{e}, \overline{s}), \forall(\overline{q}, \overline{e}, \overline{s}). \qquad (16)$$

The *incentive* constraints for a group with Pareto weights μ_1 and μ_2, which is follow directly from (13) are:

$$\sum_{c_A,q,s} \pi_\mu(c_A, q, e, s) \sum_i \mu_i u_i(c_i(c_A, \mu), e) \geq$$

$$\sum_{c_A,q,s} \pi_\mu(c_A, q, e, s) \frac{Pr(q|\widetilde{e}, k, s)}{Pr(q|e, k, s)} \sum_i \mu_i u_i(c_i(c_A, \mu), \widetilde{e}), \forall(e, \widetilde{e}) \tag{17}$$

Notice that there is an unique incentive constraint for the group regime, which reflects the fact that decisions are taken jointly. The *participation* constraint for the group regime is:

$$\sum_{c_A,q,e,s} \pi_\mu(c_A, q, e, s) u_i(c_i(c_A, \mu), e) \geq \overline{u}_i, \quad i = 1, 2 \tag{18}$$

Finally, the distribution of effort must be independent of the information set A_s, so:

$$\sum_{c_A,q} \pi(c_A, q, \overline{e}, \overline{s}) = Pr(A_{\overline{s}}) \sum_{c_A,q,s} \pi(c_A, q, \overline{e}, s), \forall(\overline{s}, \overline{e}) \tag{19}$$

The problem, optimal group arrangements are thus given by:

Program 2:

$$\max_{\pi_\mu, Pr(\mu)} \sum_{c_A,q,e,\mu} \pi_\mu(c_A, q, e, s) \cdot Pr(\mu) \cdot (q_1 + q_2 - c_A) \tag{20}$$

$$s.t.(14) \ to \ (19).$$

3.3   Comparing Regimes

As it is shown by Prescott and Townsend (2002) and Madeira and Townsend (2008), both the relative performance and the group regimes may be optimal. The relative performance regime tends to dominate for low levels of inequality and intermediate levels of surplus, while the group regime dominates for high inequality and high or low levels of surplus. In this section, we show that the optimal regime also depends on the informational state z. Proposition 1 shows that, in the symmetric case where the utility of both agents is the same, a group contract generates a higher surplus than a relative performance contract with the same utility for all agents when the common shocks are perfectly observed. Notice that a key assumption is that all output correlation comes from the common shocks ω. Conditional on ω, outputs are (ex post) uncorrelated across agents. In this sense, the result is consistent with the finding in Ramakrishnan and Thakor (1991); Itoh (1993), which show that group regimes dominate when outputs are not correlated.

Proposition 1. *Suppose there is full observability of common shocks $F_z = \{(w_1), ..., (w_S)\}$. Then, for any relative performance contract with $\bar{u}_1 = \bar{u}_2$, there exists a feasible group contract with the same utility to the agents that generate weakly higher surplus. Further, when the optimal level of effort under relative performance is not the lowest one, the feasible group contract generates a strictly higher surplus with the same utility for the agents.*

Proof. See Appendix. *Q.E.D.*

For non-symmetric cases, our numerical examples suggest that this result holds. We assume that $u(c) = c^{0.5} - e$, and admit two possible values for e (high effort and low effort): $e_h = 4$ and $e_l = 0$. We also assume two possible values for q (high output, or success, and low output, or failure): $q_h = 20$ and $q_l = 2$, and admit two possible states, 1 and 2 (so $\Omega = \{1, 2\}$) with equal probability, $Pr(\omega = 1) = Pr(\omega = 2) = 0.5$. The distribution of outputs conditional on effort is presented in Table 2, and the joint distribution of outputs in Table **??**.

Table 1: Distribution of output, $f(q|e, \omega)$.

Output level	$\omega = 1$		$\omega = 2$	
	e_h	e_l	e_h	e_l
$h = 1$	0.95	0.05	0.50	0.50
$h = 2$	0.05	0.95	0.50	0.50

Table 2: Joint distribution of outputs: $f(q_1, q2|e_1, e_2)$.

Output vector	(e_h, e_h)	(e_h, e_l)	(e_l, e_h)	(e_l, e_l)
(q_h, q_h)	0.5762	0.1487	0.1487	0.1263
(q_h, q_l)	0.1487	0.5762	0.1263	0.1487
(q_l, q_h)	0.1487	0.1263	0.5762	0.1487
(q_l, q_l)	0.1263	0.1487	0.1487	0.5762

Technically we solve two linear programming problems, *Program 1* and *Program 2* for different pairs of utility $(\bar{u}_1, \bar{u}_2)$ when agents cannot observe the common shocks, that is, $F_z = \{1, 2\}$. Figure 1 illustrates the difference between surplus under relative performance (a solution to Program 1) and the surplus under group regime (a solution to Program 2). As in Prescott and Townsend (2002) and Madeira and Townsend (2008), for some combinations of inequality and surplus (allocations with low inequality and intermediate levels of wealth) the relative performance regime generates

higher surpluses while, for others (high inequality, extreme values for wealth), the group regime is more efficient.
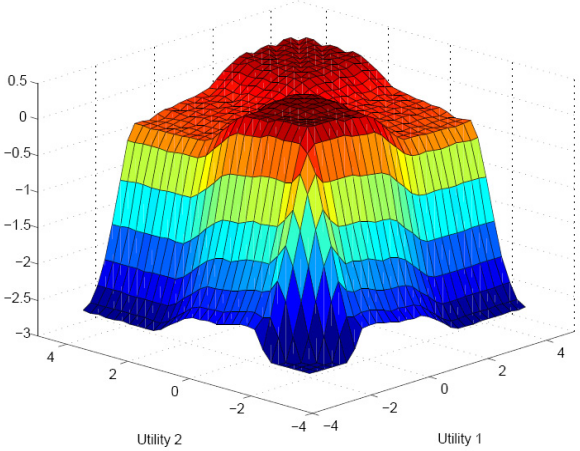

Figure 1: Surplus under *Relative Performance* minus surplus under *groups*: Coarse Filtration
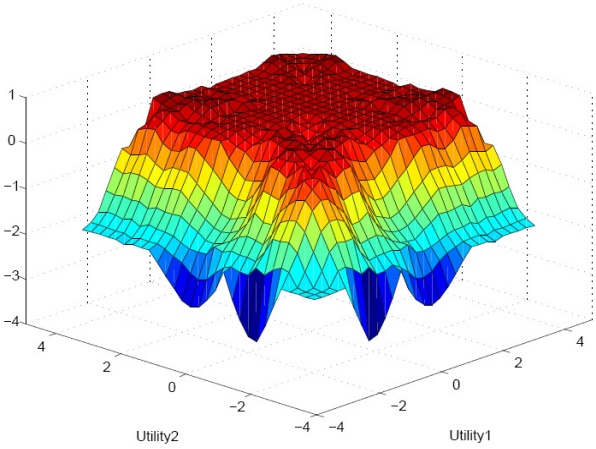

Figure 2: Surplus under *Relative Performance* minus surplus under *Groups*: fine filtration

Similarly, Figure 2 shows the difference between surplus under relative performance and the surplus under group regime when the information regime is finest, i.e., $F_{\bar{z}} = \{(1), (2)\}$. The key message is that the gains from information are unequal across regimes. In particular, in the area where relative performance dominates under z, the increment in surplus from changing to $\bar{z}$ under the group regime is larger than under relative performance. This is so that, with full information the groups regime always dominate.

The heterogeneity in the gains from information or the value of information across regimes can be seen clearly from Figure 3 and Figure 4, which show that gains from adopting a finer informational regimes under relative performance and group regimes, respectively. It is clear that, under both regimes, there is a surplus gain from a finer informational set. Notice that this example is an extreme case that makes it easy to understand the gains from information: under state 2, there is no dependency of output on effort. This means that making consumption conditional on output given state 2 does not provide any additional incentive for high effort. So, conditional on state 2 full insurance is optimal, but this can only be imposed when there is information about state.
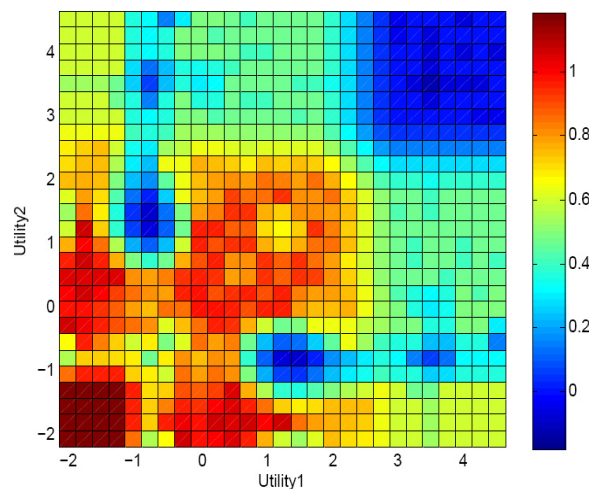

Figure 3: *Relative Performance*: surplus with fine filtration minus surplus with coarse filtration

3.4 Complete Characterization of Firms

From the analysis presented, it is possible to fully characterize the contracts under both regimes just from the type of organization, informational regime, amounts of capital and utility promises for all individuals. More formally, the set of potential firms can be defined as $B = \{b = (o, z, k, u_1, u2)\}$, where o is the type of organization (group or relative performance firm), z is the information regime, k is the amount of capital and u_i is the utility promise of agent i. As in the next section, this will define a commodity in our general equilibrium model. In other words, each individual will be (possibly randomly) assigned to a position of a firm, which implicitly includes choices of organization and investment on information.
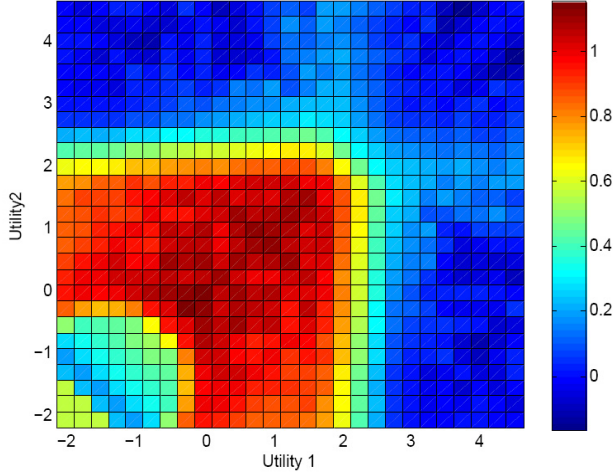
Figure 4: *Groups*: surplus with fine filtration minus surplus with coarse filtration

4   Competitive Equilibrium with Endogenous Organization

This section formulates a competitive equilibrium with endogenous organizations and informational regimes. As stated earlier, we consider only two types of firms, relative-performance and group-evaluation firms, discussed in the previous section.

4.1   Consumption Possibility Set

Let B be a finite set of all potential firms (optimal contracts with different capital input, different organizational and informational regimes, and different promised utility levels). A typical firm b specifies capital input, $k(b)$, organization form (either relative-performance or group-evaluation), information regime, $z(b)$, promised expected utility for the jth position, $v(b,j)$. In addition, each firm b is associated with a (average) surplus $S(b)$. With a continuum of firms, the average surplus from each firm is equal to the expected surplus of the firm (we here apply a law of large number).

Denote the probability of an agent type i is working at position j in firm $b \in B$ by $x^i(b,j)$. With continuum of agents, $x^i(b,j)$ can be interpreted as the fraction of agent assigned to a bundle (b,j).

As a probability measure, a lottery must satisfy

$$\sum_{(b,j)} x^i(b,j) = 1 \tag{21}$$

which is called a probability constraint.

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The consumption possibility set is then defined by

$$X^i = \left\{ x^i \geq 0 : \sum_{(b,j)} x^i(b,j) = 1 \right\} \tag{22}$$

The linearity of the constraints ensures the convexity of the consumption possibility set X^i. The preferences over X^i are defined accordingly as

$$\sum_{(b,j)} x^i(b,j) \, v(b,j) \tag{23}$$

4.2   Pareto Optimal Allocations

Each viable firm must have every positions filled. This gives us a club or matching condition (see Prescott and Townsend (2006)): for each firm b,

$$\sum_i \alpha^i x^i(b,j) = \sum_i \alpha^i x^i(b,j') = \delta(b), \ \forall j, j' \tag{24}$$

where $\delta(b)$ is the measure of active firm b.

The average consumption must be equal to the average production. In other words, the average net transfers from all firms f all types must be zero. The resource constraint is given by

$$\sum_b \delta(b) S(b) = 0 \tag{25}$$

where $S(b)$ is the average or expected surplus of firms type b.

The resource constraint for capital is given by

$$\sum_b \delta(b) [k(b) + C(b)] = \sum_i \alpha^i \kappa^i \tag{26}$$

where $C(b) = C(z(b))$ is the information cost incurring to a firm b with an informational regime F_z. This constraint states that average capital input and information cost (in unit of capital) is no larger than the average endowment of capital available. The first term on the LHS is the average capital input. The second term is the average information cost. The RHS is the average endowment of capital.

We characterize the constrained optimality using the following Pareto program. Let $\lambda^i \geq 0$ be the Pareto weight of agent type i. There is no loss of generality to normalize the weights such that $\sum_i \lambda^i = 1$. A constrained Pareto optimal allocation solves the following Pareto program.

Pareto Program

$$\max_{(x^i, \delta)} \sum_i \lambda^i \alpha^i \sum_{(b,j)} x^i(b,j) \, v(b,j) \tag{27}$$

subject to

$$\sum_{(b,j)} x^i(b,j) = 1, \ \forall i \tag{28}$$

$$\sum_i \alpha^i x^i(b,j) = \delta(b), \ \forall j, b \tag{29}$$

$$\sum_b \delta(b)[k(b) + C(b)] = \sum_i \alpha^i \kappa^i \tag{30}$$

$$\sum_b \delta(b) S(b) = 0 \tag{31}$$

Note that the Pareto program includes all incentive constraints which are embedded in the grids of potential firms.

It is clear that the objective function is linear in **x**, and thereby it is continuous and weakly concave. As discussed earlier, the feasible set X is non-empty, compact, and convex. Therefore, a solution to the Pareto program for given positive Pareto weights exists and is a global maximum. The proof of the equivalence between Pareto optimal allocations and the solutions to the program is omitted for brevity (see Prescott and Townsend (1984b) for a similar proof).

4.3 Competitive Equilibrium

Following Prescott and Townsend (2006), we define Walrasian equilibrium as follows.

Let $P(b,j)$ be the price of a bundle (b,j) or the price of the jth position in firm b. Note that the price of the capital good is 1 as it is the numeraire good. Each agent is infinitesimally small relative to the entire economy and will take all prices as given. The market-makers introduced below will also act competitively.

Workers: Each agent i, taking prices $P(b,j)$ as given, chooses $\mathbf{x}^i$ in period $t = 0$ to maximize his utility:

$$\max_{\mathbf{x}^i} \sum_{(b,j)} x^i(b,j) \, v(b,j) \tag{32}$$

subject to the probability constraint

$$\sum_{(b,j)} x^i(b,j) = 1, \tag{33}$$

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and the budget constraint

$$\sum_{(b,j)} P(b,j) x^i(b,j) \geq \kappa^i \tag{34}$$

The budget constraint (34) states that the agent sells all her endowment κ^i and uses this income to buy lotteries $\mathbf{x}^i$.

Each agent then is assigned to a firm according to the lottery by market-makers. With the continuum of agents, the probability of the lottery can be interpreted as a fraction of agents.

Market-Makers: The primary role of a market-maker is to create firms by assigning workers to organizations and generate information about aggregate states. With constant returns to scale, the profit of a market-maker must be zero and the number of market-makers becomes irrelevant. Therefore, without loss of generality we assume there is one representative market-maker, which takes prices as given.

In particular, the market-maker buys consumption goods and produce firms. The market-maker issues (sells) $y(b,j) \in \mathbb{R}_+$ units of each bundle (b,j), at the unit price $P(b,j)$. By doign so, he has to create $\delta(b)$ units of firms b. Note that the market-maker can issue any non-negative number of a bundle; that is, the number of a bundle issued does not have to be between zero and one and is not a lottery. It is the number of bundles. Let $\mathbf{y} \in L$ be the vector of the number of bundles issued.

By issuing or selling (b,j), the market-maker promises to deliver state-contingent transfers $-S(b)$ in units of consumption. The average net transfers of all firms must be no more than zero. The market-clearing constraint for the surplus is then

$$\sum_b \delta(b) S(b) \geq 0 \tag{35}$$

This transfers can be interpreted as state-contingent financial contracts among firms.

By creating $\delta(b)$ units of firms b, the market-maker must fill all positions within the firms: for each b,

$$\delta(b) = y(b,j) = y(b,j'), \ \forall j,j' \tag{36}$$

The market-maker must deliver the promised capital to each firm b, $k(b)$. In addition, it must pay for the information $C(b)$. It ensures that by buying K units of capital from individuals:

$$\sum_b \delta(b) [k(b) + C(b)] = K \tag{37}$$

The objective of a market-maker is to maximize its profit by choosing $(\delta, \mathbf{y}, K)$, taking prices $P(b, j)$ as given:

$$\max_{(\delta, \mathbf{y}, K)} \sum_{(b,j)} P(b, j) y(b, j) - K$$

subject to (35)-(37). The first term is the total revenue of the market-maker and the bracketed term denotes its total (market) cost.

Using the club or matching condition, (36), we can rewrite

$$\sum_{(b,j)} P(b, j) y(b, j) = \sum_{b} \delta(b) \left[\sum_{j} P(b, j) \right]$$

Using the above condition and (37), we can rewrite the the problem as

$$\max_{\delta} \sum_{b} \delta(b) \left[\sum_{j} P(b, j) - [k(b) + C(b)] + P_S S(b) \right] \tag{38}$$

where P_S is the Lagrange multiplier for the surplus constraint (37). This profit-maximization problem is a well-defined linear program whose (linear) constraints satisfy Slater's condition (Uzawa (1958)). In addition, the existence of an optimum to the market-maker's problem requires that for any bundle (b, j),

$$\sum_{j} P(b, j) \leq k(b) + C(b) - P_S S(b) \tag{39}$$

where it holds with equality if $y(b, j) > 0$. Here $\sum_{j} P(b, j)$ is the revenue from the sale of the memberships of firm b. This condition is in fact the necessary and sufficient condition for the saddle-point problem (38).

Define $k(b) + C(b)$ as the *input cost* of the creating firm b firm, and $-P_S S(b)$ as its *financing cost*. The market-maker considers the sum of both input cost and financing cost as its total cost for creating a firm. The optimal condition states that the market-maker will create a firm only if it does not cause a total loss. On the other hand, the revenue cannot be strictly larger than the total cost. Otherwise, the market-maker will create an unbounded amount of such firm, which cannot be in equilibrium. In addition, the total profit of the market-maker in equilibrium is zero.

Market Clearing: The market-clearing condition for capital input is

$$\sum_{(b)} \delta(b) k(b) = \sum_{i} \alpha^i \kappa^i \tag{40}$$

Similarly, the market-clearing conditions for lotteries are

$$\sum_{i} \alpha^i x^i(b, j) = y(b, j), \ \forall (b, j) \tag{41}$$

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Definition 1. A competitive equilibrium is a specification of allocation $(\mathbf{x}, \mathbf{y}, \delta)$, and the price of a position j in a firm b, $P(b, j)$, such that

(i) for each i, $\mathbf{x}^i \in X^i$ solves (32) subject to (34), taking prices $P(b, j)$ as given,

(ii) for the market-maker, $\{\mathbf{y}, \delta, P_S\}$ solves (38), taking prices $P(b, j)$ as given,

(iii) markets for capital input and lotteries clear, i.e. (40)-(41) hold,

5 Existence and Welfare Theorems

As in the classical general equilibrium model, the economy is a well-defined convex economy, i.e., the commodity space is Euclidean, the consumption set is compact and convex, the utility function is linear. As a result, the first and second welfare theorems hold, and a competitive equilibrium exists. In particular, this section proves that the competitive equilibrium is constrained optimal and any constrained optimal allocation can be supported by a competitive equilibrium with transfers. Then, we will use Negishi's method to prove the existence of a competitive equilibrium.

The standard contradiction argument will be used to prove the following first welfare theorem. We also assume that there is no local satiation point in the consumption set.

Assumption 1. *For any* $\mathbf{x}^i \in X^i$, *there exists* $\tilde{\mathbf{x}}^i \in X^i$ *such that*

$$\mathscr{U}^i\left(\tilde{\mathbf{x}}^i\right) > \mathscr{U}^i\left(\mathbf{x}^i\right) \tag{42}$$

where $\mathscr{U}^i\left(\mathbf{x}^i\right)$ *is the expected utility of agent i derived from allocation* $\mathbf{x}^i$.

This assumption is easily satisfied using reasonable specifications of the grid of promised utility. For example, with a strictly increasing utility function, if we include a very large promised utility into the grid (larger than what can be attained with endowments), then the local nonsatiation assumption will be satisfied.

Theorem 1. *With local nonsatiation of preferences (Assumption 1), a competitive equilibrium allocation is (constrained) Pareto optimal.*

The Second Welfare theorem states that any Pareto optimal allocation, corresponding to strictly positive Pareto weights, can be supported as a competitive equilibrium with transfers, precisely defined later. The standard approach applies here. In particular, we will first prove that any constrained optimal allocation can be decentralized as a compensated equilibrium. To prove this

result, we show that a solution to the Pareto program is also a solution to expenditure minimization and profit maximization problems in equilibrium. Since all the optimization problems are well-defined concave problems, the Kuhn-Tucker conditions are necessary and sufficient. Hence, we will show that the Kuhn-Tucker conditions of Pareto program are equivalent to the ones of consumers' and intermediary's problems in equilibrium. In addition, the resource constraints in the Pareto programs are equivalent to the market-clearing conditions in equilibrium. Then, we will use a standard cheaper-point argument (see Debreu (1954)) to show that any compensated equilibrium is a competitive equilibrium with transfers.

Theorem 2. *Any Pareto optimal allocation corresponding with strictly positive Pareto weights $\lambda^h > 0, \forall h$ can be supported as a competitive equilibrium with transfers.*

We use Negishi's mapping method (Negishi (1960)) to prove the existence of competitive equilibrium. The proof benefits from the second welfare theorem. Specifically, a part of the mapping applies the theorem in that the solution to the Pareto program is a competitive equilibrium with transfers. We can show that a fixed-point of the mapping exists and it represents a competitive equilibrium without transfers.

Theorem 3. *For any positive endowments, a competitive equilibrium exists.*

6 Numerical Examples

This section presents numerical examples of competitive equilibria, and discusses the interplay between organization, investment in information, and inequality. In particular, we will solve the Pareto Program whose solution is a (constrained) Pareto optimal, and employ the second welfare theorem to discover the corresponding competitive equilibrium. The equilibrium prices are computed using the following relationships:

$$P(b, j) = \frac{\mu_m(b, j)}{\mu_k} \tag{43}$$

$$P_S = \frac{\mu_S}{\mu_k} \tag{44}$$

where $\mu_m(b, j), \mu_k, \mu_S$ are the Lagrange multipliers with respect to the matching constraint (29), the resource constraint (30), and the surplus constraint (31), respectively. Accordingly, the capital endowments that support the competitive equilibrium allocation can be derived from the budget constraint:

$$\kappa^i = \sum_{(b,j)} P(b, j) x^i(b, j) \tag{45}$$

Similarly to examples in Section 3.3, the following numerical examples assume that $u(c) = c^{0.5} - e$, and admit two possible values for e (high effort and low effort): $e_h = 4$ and $e_l = 0$. We also assume two possible values for q (high output, or success, and low output, or failure): $q_h = 20$ and $q_l = 2$, and admit two possible states, 1 and 2 (so $\Omega = \{1, 2\}$) with equal probability, $Pr(\omega = 1) = Pr(\omega = 2) = 0.5$. The production technology is given by $\tilde{f}(q+k|e, \omega, k) = f(q|e, \omega)+k$, where $f(q|e, \omega)$ is defined according to the distribution in Table 2 as in Section 3.3. There are two types of agents, type 1 and type 2. In addition, we assume that, at a cost of $C(2) = 0.96$, every ω will be observed.

Figures 5 displays the probabilities of relative performance assignment for different combinations of aggregate endowment and Pareto weights. It reveals that the regime dominance profile changes when the possibility of investment in the observability of common shocks is introduced.

As in the standard multi-agent moral hazard model, relative performance tends to dominate for low inequality and intermediate wealth levels (e.g., Prescott and Townsend, 2002). However, the area of relative performance dominance is not continuous when it is possible to observe the common shocks. In particular, there is a region with group domination between two separate areas with relative performance domination. This strip represents an area where information is very valuable. Indeed, as it is shown in Figure 6, the probability of investment on information is high at this area relative to its neighborhood.
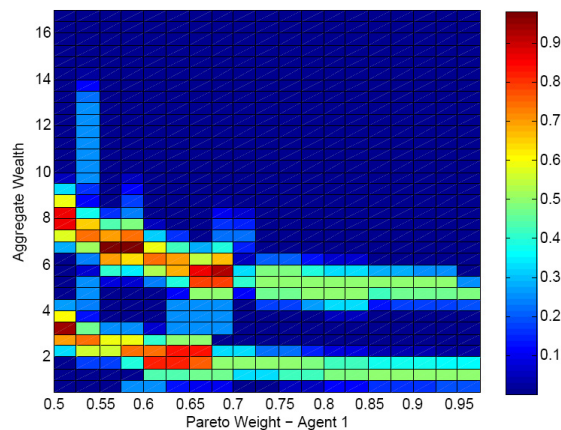

Figure 5: Probability of assignment to the *Relative Performance* regime.

Tables I to IV describe examples of competitive equilibria for different initial endowment levels, Pareto weights of types and fraction of types in the population. Each column corresponds to a position (in the 7th line) for a given type (in the 8th line) in an organization (indexed in line 2)
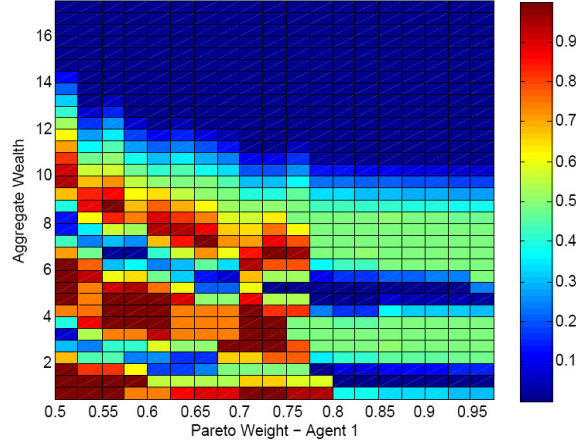
Figure 6: Probability of investment on information.

that is chosen with positive probability. This probability (that the type in line 8 is assigned to the position in line 7 of the organization indexed in line 2) is presented in line 1. Lines 3 and 4 describes the utility of the first and second positions in each contract and lines 5 and 6 are indicators of organizational arrangement and investment in observability of common shocks. More specifically, in line 5, "1" indicates the group regime, while "0" indicates the relative performance regime. In line 6, "1" indicates that there is investment in observability and "0" indicates no investment in observability. Line 9 shows the price of the contract indexed in line 2 for the position in line 7. Just below the tables, w1 and w2 denote the initial capital endowments os types 1 and 2.

Table I

	K=8	α1=0.5	λ1=0.5	Baseline	C(2)=0.957			
Probability	0.20159	0.048411	0.20159	0.048411	0.22923	0.020768	0.22923	0.020768
Contract Number	435	3166	435	3166	435	3166	435	3166
Utility 1	1.1084	1.3619	1.1084	1.3619	1.1084	1.3619	1.1084	1.3619
Utility 2	1.1084	1.3619	1.1084	1.3619	1.1084	1.3619	1.1084	1.3619
Indicator Gr	0	1	0	1	0	1	0	1
Indicator Inf	0	1	0	1	0	1	0	1
Position	1	1	2	2	1	1	2	2
Type	1	1	1	1	2	2	2	2
Price	7.7417	9.6088	7.7417	9.6088	7.7417	9.6088	7.7417	9.6088

w1=4.0516   w2= 3.9484

Table II describes the symmetric case where both types of agent have the same Pareto weight. Notice that both kinds of organization are observed with positive probability. In this example, groups are only observed in organizations that investment on observability of common shocks. But this is not always the case. Table II presents an example where Pareto weights are different across

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types (0.75 for agent 1 and 0.25 for agent 2). Individuals are always assigned to groups, and there is no investment in information. Notice that, as in Prescott and Townsend (2006), the prices of some positions are negative. An amount is received by individuals that join those positions, and this amount can be interpreted as wage. Also, the wealth (or endowment) of individuals of type two is negative, so they can be thought as indebted agents. Table III departs from the environment in Table II but the fraction of individuals of type 1 is increased (to 0.75). Now, both types of regime coexist and also, groups with and without investment on observability of common shocks are observed. This examples shows that different organizational arrangements and levels of investment in observability can be observed simultaneously.

Table II

	K=8	α1=0.5	λ1=0.75	Baseline	C(2)=0.957			
Probability	0.20647	0.043528	0.20647	0.043528	0.20647	0.043528	0.20647	0.043528
Contract Number	1478	1508	1130	1131	1130	1131	1478	1508
Utility 1	2.3757	2.6292	-0.66579	-0.66579	-0.66579	-0.66579	2.3757	2.6292
Utility 2	-0.66579	-0.66579	2.3757	2.6292	2.3757	2.6292	-0.66579	-0.66579
Indicator Gr	1	1	1	1	1	1	1	1
Indicator Inf	0	0	0	0	0	0	0	0
Position	1	1	2	2	1	1	2	2
Type	1	1	1	1	2	2	2	2
Price	17.389	19.359	17.389	19.359	-1.7322	-1.7322	-1.7322	-1.7322

w1=8.8861   w2= -0.66609

In table IV we depart from the environment in table I and decrease the correlation of outputs across individuals with high effort from 0.32 (the value obtained with the specification in section 3.3) to 0.12[5]. Surprisingly, as common shocks become less important and correlation diminishes, investment on observability increases (it is observed in all contracts of Table IV). This happens because correlation makes relative performance useful for it's capacity of extracting information about one agent's effort from the other agent's input. Lower correlation diminishes this power, so the surplus with relative performance becomes lower, increasing the difference between the full information surplus and the coarse filtration relative performance surplus.

7   Conclusion

This paper extends the multi-agent moral hazard model of Holmstrom and Milgrom (1990) in two dimensions: (i) we add costly observability of common shocks, and (ii) we embed it into a Walrasian

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[5]In order to generate a lower correlation, we decreased the probability of high output given high effort with $\omega = 1$ and increased it with $\omega = 1$. The unconditional distribution of outputs was then kept unchanged, but the distribution of outputs conditional on ω became closer, so the output correlation across agents diminished

Table III

	K=8	α1=0.75	λ1=0.75	Baseline	C(2)=0.957			
Probability	0.078787	0.29821	0.078787	0.29821	0.046213	0.078787	0.046213	0.078787
Contract Number	1478	3166	1130	3166	218	1130	218	1478
Utility 1	2.3757	1.3619	-0.66579	1.3619	-0.66579	-0.66579	-0.66579	2.3757
Utility 2	-0.66579	1.3619	2.3757	1.3619	-0.66579	2.3757	-0.66579	-0.66579
Indicator Gr	1	1	1	1	0	1	0	1
Indicator Inf	0	1	0	1	0	0	0	0
Position	1	1	2	2	1	1	2	2
Type	1	1	1	1	2	2	2	2
Price	17.384	9.6088	17.384	9.6088	-1.7273	-1.7273	-1.7273	-1.7273

w1=8.4318   w2=-0.43182

Table IV

	K=8	α1=0.5	λ1=0.5	Lower Corr	C(2)=0.957			
Probability	0.20401	0.04599	0.20401	0.04599	0.22733	0.02267	0.22733	0.02267
Contract Number	3135	3166	3135	3166	3135	3166	3135	3166
Utility 1	1.1084	1.3619	1.1084	1.3619	1.1084	1.3619	1.1084	1.3619
Utility 2	1.1084	1.3619	1.1084	1.3619	1.1084	1.3619	1.1084	1.3619
Indicator Gr	1	1	1	1	1	1	1	1
Indicator Inf	1	1	1	1	1	1	1	1
Position	1	1	2	2	1	1	2	2
Type	1	1	1	1	2	2	2	2
Price	7.7437	9.6104	7.7437	9.6104	7.7437	9.6104	7.7437	9.6104

w1=4.0435   w2= 3.9565

equilibrium model. We present theoretical and numerical results showing that both regimes can be optimal, but information about the common shocks benefits cooperative groups more. These results suggest that the emergence of cooperative behaviors may be related with technological advances, e.g., a lower cost for acquiring information. The relationship between organization and correlation in outputs is subtler: when organizational choice and investment on information are jointly determined, the prevalence of individualistic competitive arrangements is not necessarily higher when outputs are more correlated (contrast to Ramakrishnan and Thakor, 1991; Itoh, 1993).

Numerical computation of competitive equilibria reveals that diversity of organizational formats exists in equilibrium. The possibility of investment in observability affects significatively the profile of organization across levels of wealth and inequality: groups emerge at low levels of inequality and intermediate levels of wealth, in an region where information about common shocks is valuable (this is different from the result in Prescott and Townsend, 2002, which suggest that relative performance would dominate).Interestingly, our results suggest that only collective organizations may invest in information about the common shocks, especially for groups in the area of low inequality among agents. All these patterns can potentially be confronted to data.

The current framework accept a number of extensions. A dynamic version of the model could be informative about co-movements and transitions, and also generate patterns of evolution of organizational form as agents learn about the environment and the production process. In addition, we are working on a model with optimal size of organizations and it's interplay with availability of information about shocks.

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A   Proof of Proposition 1

Proposition 1 *Suppose there are two possible levels for effort and output. Suppose also that $F_z = w_1, ..., w_S$. Then, any relative performance arrangement with $u_1 = u_2$ generates a surplus that*

is not higher than some feasible contract under groups that generate the same amount of utility to the agents. Further, when the optimal level of effort under relative performance is the high one, there is a group contract that generates a higher surplus with the same utility for agents.

Proof. We prove the result for the case with 2 levels of effort and two levels of output, but it readily generalizes for the case with multiple outputs and levels of effort. For expositional convenience, let p_s be the probability of state s and define p_{hs}^l as the probability of low output when state is s and effort is high. Define accordingly p_{ls}^l, p_{ls}^h and p_{hs}^h. Take some relative performance arrangement. Since $u_1 = u_2$, one individual does not inform about the other, and $u(c)$ is concave, we can assume, without loss of generality, that there is a symmetric solution with the consumption of agent i, $c_i = c_s^q$ in the case of individual output q and state s. Now, let us consider that for each state s, we determine the following group arrangement: when the state is s, and the output of both individuals is high, $c_1 = c_2 = c_s^h$ with probability 1. When the state is s and the output of both individuals is low, $c_1 = c_2 = c_s^l$ with probability one. When only one individual has high output, $c_1 = c_2 = c_s^m$ with probability one, where $u(c_s^m) = 0.5u(c_s^l) + 0.5u(c_s^h)$. This implies that the weighted utility of a group with μ equal to $\{0.5, 0.5\}$ with the recommended amount of effort is:

$$U_{\mu=0.5}(e_h, e_h) = \sum_s p_s p_{hs}^h p_{hs}^h u(c_s^h) + 2\sum_s p_s p_{hs}^h p_{hs}^l u(c_s^m) + \sum_s p_s p_{hs}^l p_{hs}^l u(c_s^h) - e_h =$$
$$\sum_s p_s p_{hs}^h p_{hs}^h u(c_s^h) + 2\sum_s p_s p_{hs}^h p_{hs}^l [0.5u(c_s^l) + 0.5u(c_s^h)] + \sum_s p_s p_{hs}^l p_{hs}^l u(c_s^h) - e_h =$$
$$\sum_s p_s p_{hs}^h p_{hs}^h u(c_s^h) + \sum_s p_s [p_{hs}^h (1 - p_{hs}^h) u(c_s^l) + p_{hs}^l (1 - p_{hs}^l) u(c_s^l)] + \sum_s p_s p_{hs}^l p_{hs}^l u(c_s^h) - e_h =$$
$$\sum_s p_s p_{hs}^h u(c_s^h) + \sum_s p_s p_{hs}^l u(c_s^h) - e_h,$$

the same as in the *relative performance* regime. The weighted utility for the group of having one individual doing an alternative amount of effort is given by:

$$U_{\mu=0.5}(e_h, e_l) = \sum_s p_s p_{hs}^h p_{ls}^h u(c_s^h) + \sum_s p_s p_{hs}^h p_{ls}^l u(c_s^m) +$$
$$\sum_s p_s p_{hs}^h p_{ls}^h u(c_s^h) + \sum_s p_s p_{hs}^h p_{ls}^l u(c_s^m) + \sum_s p_s p_{hs}^l p_{ls}^h u(c_s^m) +$$
$$\sum_s p_s p_{hs}^l p_{ls}^l u(c_s^h) - \frac{e_h + e_l}{2} =$$
$$\sum_s p_s p_{hs}^h p_{ls}^h u(c_s^h) + \sum_s p_s p_{hs}^h p_{ls}^l (1/2)[u(c_s^h) + u(c_s^l)] +$$
$$\sum_s p_s p_{hs}^l p_{ls}^h (1/2)[u(c_s^h) + u(c_s^l)] + \sum_s p_s p_{hs}^l p_{ls}^l u(c_s^h) - \frac{e_h + e_l}{2} =$$
$$\sum_s p_s p_{hs}^h [p_{ls}^h u(c_s^h) + p_{ls}^l (1/2)u(c_s^h) + p_{ls}^l (1/2)u(c_s^l)] +$$
$$\sum_s p_s p_{hs}^l [p_{ls}^h (1/2)u(c_s^h) + p_{ls}^h (1/2)u(c_s^l) + p_{ls}^l u(c_s^l)] - \frac{e_h + e_l}{2} =$$
$$\sum_s p_s p_{hs}^h [(1/2)p_{ls}^h u(c_s^h) + (1/2)u(c_s^h) + p_{ls}^l (1/2)u(c_s^l)] +$$
$$\sum_s p_s p_{hs}^l [p_{ls}^h (1/2)u(c_s^h) + (1/2)u(c_s^l) + (1/2)p_{ls}^l u(c_s^l)] - \frac{e_h + e_l}{2} =$$
$$\sum_s \frac{p_s}{2} [p_{ls}^h u(c_s^h) + p_{ls}^l u(c_s^l) + p_{hs}^h u(c_s^h) + p_{hs}^l u(c_s^l)] - \frac{e_h + e_l}{2}.$$

The last equality comes from the fact that $p_{hs}^l + p_{hs}^h = 1$. In the relative performance regime, the gain from higher consumption given high effort perfectly compensates the higher disutility of high effort, so:

$$\sum_s p_s[p_{ls}^h u(c_s^h) + p_{ls}^l u(c_s^l)] = \sum_s p_s[p_{hs}^h u(c_s^h) + p_{hs}^l u(c_s^l)] + e_l - e_h.$$

Substituting above, we have:

$$U_{\mu=0.5}(e_h, e_l) = \sum_s p_s[p_{hs}^h u(c_s^h) + p_{hs}^l u(c_s^l) + \tfrac{e_l - e_h}{2}] - \tfrac{e_h + e_l}{2} = \sum_s p_s[p_{hs}^h u(c_s^h) + p_{hs}^l u(c_s^l)] - e_h,$$

which implies that the group has no gain in adopting low effort for one individual. Straightforward analogous procedure reveals that the group also does not have any incentive to adopt low effort by both individuals. Clearly, the participation, mother nature and compatibility with state distribution constraints are valid. However, since u is strictly convex, $c_s^m < (c_s^h + c_s^l)/2$, which implies that the group regime produce the same utility pair with lower cost, and higher surplus. $Q.E.D.$