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QUANTITY-BEFORE-PRICE AUCTION: EVALUATING THE PERFORMANCE OF THE BRAZILIAN EXISTING ENERGY MARKET

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Quantity-before-Price Auction: Evaluating the Performance of the Brazilian Existing Energy Market

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WORKING PAPER

Abstract

In this paper we study the outcome of the six first "existing energy" auctions in Brazil. Under the current regulatory regime, the bulk of the trade between existing generators and distribution companies is done through bilateral eight-year contracts, negotiated in auctions organized by the regulatory agency. These so-called existing energy auctions follow a specific rule that we call quantity-before-price auction. In this paper we develop an auction model to study this type of game and take the model to the data. The structural model enables us to estimate the opportunity cost of selling energy for each generator, which is the price they expect to obtain in the free market in the future.

We find that, while auction prices have steadily risen, the opportunity cost has increased even faster over time, indicating that the generators anticipate a faster increase in energy prices than what is measured by auction prices.

Keywords: Auctions, Electricity, Brazil. JEL Codes: D44, L94.

1 Introduction

This paper develops a model of the quantity-before-price auction and uses it to analyze the Brazilian existing energy market. Most of the electricity trade in Brazil is done through bilateral contracts that are bought at auctions run by the regulatory agency, that acts as a single buyer on behalf of the distribution companies (discos). Generators can choose between selling their energy on the auctions or negotiate it on the so-called 'free market', composed of large individual consumers and traders. The price of the free market is therefore the opportunity cost of the electricity being negotiated in the auction. We develop an auction model that takes into account the institutional framework of the Brazilian market, and take this model to data. The structural model enables us to estimate the opportunity cost of the electricity traded which corresponds to the unobservable price of the free market. The results show that the opportunity cost of the water increased steadily across the auctions, indicating that the market indeed expects high prices in the future.

We find evidence of prices above cost in auctions for contracts starting in 2006, 2007 and 2008, with mark ups reaching up to 66% on the 2006's auction, and virtually no evidence of market power on the other auctions. We conjecture that large amounts of non-contracted capacity on the generators side increased competition on the first auction, with contracts starting in 2005, driving the mark up down. Despite the reduction in non-contracted capacity across the auctions, the decreasing in the quantity demanded on the last auctions had the effect to once again increase competition among sellers.

The electricity trade happens through medium and long term contracts of energy supply, traded both at the regulated market — the auctions — and the free market. The demand in the regulated market consists of the discos' demand, since they must purchase all their electricity needs in the auctions, and some large users. This obligation guarantees that around 75% of the electricity trade happens in this environment. The sellers are generators and trading companies. The most common product traded in this market are eight years contracts of electricity supply, starting on a pre-determined year in the future. Sometimes shorter term contracts are also negotiated.

The regulatory agency acts as a single buyer of the electricity. The discos declare to the regulator the amount of energy they need to buy. The regulator then runs the auction with the generators as suppliers. The goal is to buy the amount needed by the distributors, but the regulator may decrease the demand to induce competition in the auction. The total energy bought is pooled together and allocated to the discos proportional to their declared needs and at the average price paid to suppliers.

The free market is a bilateral market where agents are free to trade contracts of energy supply. There are no special rules here, only that the contracts must be registered with the ISO for balance control, as it is usual in electricity markets. Generators are free to negotiate their electricity in any of the two markets.

The auction mechanism consists of two stages. In the first one, henceforth called the quantity phase, bidders privately choose in a descending price auction the quantity they are willing to supply at the current price. After the aggregate quantity crosses an unknown (to the bidders) threshold, this phase stops and the game move to the second phase. On the second phase, henceforth called price phase, bidders choose in a one shot game the price at which they want to sell the quantity previously defined on the first phase. We call this auction a *quantity-before-price* auction.

Correia, Melo, and da Costa (2006) provide a discussion of how this choice of auction design was made. The two-phase arrangement, with a first-price auction following a English auction, is reminiscent of the rule proposed by Klemperer (1998) to sell US airwave spectrum auctions.

We analyze data from six consecutive auctions, which traded contracts of eight years supply starting in 2005, 2006, 2007, 2008, 2009 and 2007 (there were two auctions selling contracts with 2007 as the starting year).

Using the first order condition for optimal bidding in the second phase of the auction, we derive an expression for the unobserved opportunity cost of the sellers. Using the bidding data we estimated the distribution of bids conditional on agents characteristics. We used information about the quantities to calculate the expected quantities bidders use to calculate the probability of winning. With these two pieces of information and the first order condition we can compute the opportunity costs of the generators.

We explore the fact that the auctions are sequential and introduce learning into the model. The players have some prior uncertainty about how the other players will bid. This uncertainty is represented by a stochastic term, for which bidders update their beliefs after seeing the data and learn about its distribution.

The paper is organized as follows . After this introduction the next section describes the Brazilian electricity market. Section 3 describes the auctions results and discuss the question of whether the state owned companies bid lower prices. Section 4 presents a theoretical model of the auction. Section 5 describe the learning process. Section 6 explains the method we used to estimate the structural model developed in section 4. Section 6 presents the results from both reduced form and structural estimation methods. The last section concludes.

2 The Electricity Market in Brazil

This section provides a brief background on the institutional environment which lead to the reforms that created the existing energy auctions. A more detailed account can be found in Lock (2005) and Dutra and Menezes (2005).

The reforms in the nineties transformed a system with state owned vertical integrated utilities into a (partially) private owned system with generation, transmission and distribution independent of each other. An independent regulatory agency was created, ANEEL (National Agency of Electric Energy), to give credibility to the regulatory process and attract private capital to the sector.

The reforms started in 1995 with the first privatizations and continued during the late 1990's and early 2000's. ANEEL was created in 1996, with the privatizations already on its way, and it is responsible for the regulation of the operation and expansion of the three segments: electricity generation transmission and distribution. A wholesale market for electricity was created in 2000.

One important aspect of the reforms of the 1990's was the creation of a market for bilateral contracts between generators and consumers, mainly distribution companies (discos) and large users. The two sides were free to trade long term contracts of electricity supply. An important distinction of the Brazilian market to other market arrangements is that competition exists at the contracting level but not at the production level, with the actual production of electricity being a centralized decision (against the more popular design of competition at the production level, like California or UK).

An important event was the energy crisis of 2001. An exceptionally dry rain season in the first months of 2001 left the reservoirs in the Southeast and West Central regions with critically low levels of water. The water shortage lead to a mandatory national rationing of electricity consumption, extremely high spot prices and non clearance of the wholesale market, culminating in litigation between several generators and distributors and eventually a federal intervention in the wholesale market.

The crisis of 2001 raised severe criticism about the functioning of market institutions, and had a significant negative impact on the public opinion's view of the reforms of the past decade. The government argued that the crisis was due to insufficient rain in the summer of 2001, while critics of the reforms charged that it was a consequence of misdesigned institutions that were not able to attract enough investment to the sector.

One characteristic of the electricity market is that supply must meet demand continuously. This requires a tight control on the amount being produced at any instant in time, which in part explains the high degree of regulation of even the most pro-competitive electricity markets. Another characteristic is that it cannot be (economically) stored. Therefore, the installed production capacity must be enough to produce up to demand at its peak, otherwise the system collapses.

One of the reasons for the 2001 crisis was the inability of the bilateral contract market to attract investments soon enough for these investments to mature at the time of the increase in demand.[1] New generators entered the market in the years that anticipated the crisis (Moita, 2006), but they effectively started operation only some years after 2001. As a result, a perception emerged that the bilateral market was not capable of coordinating the entry of new generators to keep up with the future demand.

As a result the federal government decided to create a more centralized mechanism to trade these contracts. The model adopted was a single buyer model where long term contracts are purchased through an auction from the producers by the government and sold to distributors.

2.1 The Industry

The Brazilian generation industry is somewhat concentrated. The maximum generation capacity of the ten largest firms is shown on table 1. They have altogether 67% of total capacity of the Brazilian system, what indicates a high degree of concentration. Among those ten, Chesf, Furnas, Eletronorte and Itaipu belong to the federal government through the holding Eletrobras Group, which accounts for 36% of the total capacity.

The high degree of industry concentration led the regulatory agency ANEEL to adopt an auction model in which the information about the auction that bidders can access during the auction is minimal. During the first part of the auction the sellers do not know the aggregate supply or the total demand, as a way to try to avoid collusion.

[1] Investments in hydroelectric plants take about five years to start producing.

Table 1: Ten Largest Generating Firms

Firms	**Gen Capacity** (MW)	**Gen Capacity** (% of total gen)
CHESF	10615	10.9
FURNAS	9656	10.0
ELETRONORTE	8046	8.3
CESP	7455	7.7
CEMIG	6782	7.0
TRACTEBEL	6515	6.7
ITAIPU	6300	6.5
COPEL	4545	4.7
AES TIETE	2651	2.7
DUKE	2299	2.4
ELETROBRAS GROUP	34617	35.7
TOTAL 10 LARGEST	64864	66.9
TOTAL GEN CAPACITY	96971	100.0

The generation capacity of the plants, as shown in table 1, is not the maximum amount a firm can sell in the auction. First, hydropower plant generators can only contract up to what is called their 'assured energy'. It is a percentage of the plant's total capacity and it is defined as the amount that can be supplied over the next years with a 5% percent maximum chance of not having enough water to deliver it. It depends on the distribution of rainfall in the specific location the plant is located, among other things. The second reason is that generators may have part of their capacity already contracted before the auction.

3 Outcomes of the auctions

Figure 1 shows the results of the auctions for eight years contracts of electricity supply starting in 2005, 2006, 2007, 2008 and 2009. The graph depicts the supply-demand schedules (R$/MWh) for each of the five auctions.[2] The supply schedule of the winning bids shift upward and to the left across the auctions. It implies in higher prices and smaller quantities for the later auctions.[3]

The generators can choose between selling in the auction or not. If they do not sell on the auction, they can negotiate this electricity on the free market or sell it at the spot price. Therefore, the price from the free market and the spot price represent the opportunity cost for the electricity traded through the auctions. Following this reasoning, one can think of the auction prices as an

[2] Demands are assumed to be inelastic. This is not strictly true; discos are require to submit inelastic demand schedules to the regulator, but the regulator may withhold demand if prices are deemed too high.

[3] The total demand also decreased for each auction. Since the first contracts of electricity supply signed at the time of the privatization and creation of the electricity market were expiring in 2004 2005 and 2006, there were a large amount of the electricity to be contracted in these years, what explains the larger volume of electricity negotiated for the contracts initiating in 2005 and 2006. Since the contracts overlap, from 2007 on only the expected increase in demand were negotiated in the auctions for contracts starting on these years.

expectation of these future prices. Figure 2 shows the correlation between the spot and auction prices. The higher prices for the contracts starting in 2006 represent the expectation for the also higher spot prices of that year.

The spot price is the Lagrange multiplier of the feasibility constraint of the optimal dispatch problem solved by the operator of the system. It corresponds to the reported marginal cost of the last generator to be dispatched. While only a small fraction of the overall load is traded at this price, it is an appropriate indicator of the scarcity of energy at a specific point in time.

Figure 1 depicts increasing auction prices from 2005 on, as shown by the movements to the left of the supply schedule, which represents increasing expected spot prices.

3.1 Did the Public Firms Bid Differently than the Private?

This question comes from the anecdotal view that the public firms sell energy at a lower price (ironically called patriotic prices) in the auctions. In fact, there is some evidence that they did, at least in the first four auctions, but it also depends on how one defines public firms.

Since the reforms the sector undergone during the 1990's were conduced by the government at the federal level, which is also the responsible for the sector's regulation, any dysfunction of the electricity industry, such as price spikes or black-outs, is blamed on the federal government. The political cost of obtaining high electricity prices on the auctions could explain a more aggressive bidding strategy by the state owned companies. Hence, one would expect the public companies owned by the federal government to have a political motive to bid lower than the rest.

Table 2 shows the results of OLS regressions of the price bids on some covariates. The important variables to answer the question above are pub. enterpr. and fed. enterpr., with the former being a dummy variable for any firm that has either federal or state governments as the owner of the majority of the shares and the latter being a dummy for those companies that have the federal government as the owner of at least 50% of the firm's shares. The results show that if we define state enterprises as the ones owned by any level of government (state or federal) there is no statistical differences between public and private. The coefficients are not negative in any case, and in fact it is positive and significant (at 15% level) if we look at the results of all auctions in our sample, as show in column 5 of the Table 2.

However, when we consider only the federally owned companies the results change. If we look at the result of whole sample on column 3, we have a not statistically different than zero coefficient. But if we consider only the first three (column 2) or four auctions (column 6) we have a significant negative coefficient for the fed enterpr variable.

The results, however, are not conclusive. What we know is that Chesf, Furnas and Eletronorte bid lower prices in the first auctions, but abandon this strategy on the more recent ones. More information is needed to know if it was a case of 'patriotic prices', with the firms bidding low to lower the market price for final consumers for political reasons, and running out of available capacity to be able to manipulate the price in the subsequent auctions; or if it is a case of winner's curse, with an excessively aggressive bidding strategy by the

Table 2: OLS Price Regressions

	First 3 Auctions		All Auctions		First 4 Auctions
	federal co.	*public co*	*federal co.*	*public co.*	*federal co.*
constant	57.18	56.83	57.34	55.98	57.19
	(41.66)	(36.22)	(20.54)	(19.74)	(45.35)
quantity	0.00	0.00	0.00	0.00	0.00
	(1.52)	(0.02)	(0.09)	(-0.23)	(1.21)
state enterpr.		1.31		3.36	
		(0.85)		(1.65)	
fed. enterpr.	-3.55		0.43		-2.01
	(-1.99)		(0.17)		(-1.50)
D2006	9.26	8.83	7.96	7.56	9.15
	(5.91)	(5.29)	(2.61)	(2.53)	(6.36)
D2007	18.47	16.91	26.71	25.79	17.96
	(9.67)	(8.70)	(7.50)	(7.62)	(10.40)
D2008			25.19	25.07	25.94
			(7.00)	(7.30)	(15.93)
D2009			37.10	36.31	
			(10.27)	(10.41)	

Numbers in parentheses are t ratios.

firms at the beginning followed by regret and more conservative bidding in the subsequent auctions.

One interesting feature is that prices are not correlated with quantity. We use this result as a simplifying assumption when formulating the structural bidding model. The increase in the year effects can be understood by looking at Figure 1. It is due to the upward shift in the supply schedule of the generators that happened toward the more recent auctions.

4 The Quantity-before-Price Auction

In this section, we develop a theoretical model for the auction game used in the existing energy market. This institution has also been describe and investigated in Dutra and Menezes (2005). Here, the focus will be to derive from the theoretical model a method that will make it possible to draw empirical inferences about competition and implicit generator costs, as it will be seen below.

We consider a game where N suppliers (generators) compete to provide a homogeneous, divisible good to a single buyer. Generator i has a technology represented by a "supply function" $S_i(p) = mc_i^{-1}(p)$, where mc_i is i marginal cost curve. The buyer seeks to procure a fixed amount Q^* of the good.

Most of the time we will specialize to the case where each supplier has an inverted L-shaped cost structure, with a fixed capacity $\bar{q}_i$ and a constant marginal cost c_i for any quantity supplied up to $\bar{q}_i$. We abuse terminology and call this situation "constant marginal costs".

For simplicity, we will assume that bidders make their decision maximizing profits in each auction in isolation. In reality, since contracts from separate auctions traded in the Brazilian existing energy market overlap and the the auctions sometimes occur simultaneously, different auctions are strategically interrelated; see Dutra and Menezes (2005).

The auction runs in two phases, the quantity phase and the price phase. The quantity phase is a descending uniform price auction. Specifically, the auctioneer starts by announcing an initial high price, a quantity threshold $\bar{Q} \geq Q*$ and a price decrement Δ. Bidders submit quantity bids. (A bid q_i at price p is a promise to deliver q_i at a price of at most p.) If the sum of the submitted bids is less than $\bar{Q}$, this phase ends; otherwise, the posted price is decreased by Δ, and quantity bidding continues.

The price phase is a one-shot discriminatory price auction. Each bidder is required to post a single price bid to supply the quantity offered in the second-to-last round of the quantity phase. The current price in the second-to-last round is the reserve price in the price phase (thus, the quantity bids in the first phase are firm obligations). The winners are the generators that post the lowest prices, up to the point where the initial demand is satisfied. Ties are broken randomly.

Finally, we assume that bidders learn through the auctions. Given an initial level of uncertainty on the first auction, players learn how to form expectations about the other bidders' price bid. This is equivalent to assume that the bidders in our sample are bayesians, and after seeing the data use Baye's rule to update their beliefs about the opponents' bids in the next auction. This assumption is motivated by the fact that we are observing a market in its infant periods, with substantial initial uncertainty about the competitive environment.

4.1 Competitive Behavior

We say a generator behaves competitively if it does not purposefully restricts its supply in order to keep prices high; in the context of this auction, by competitive behavior we mean the following: suppose in a given stage of the quantity phase of the auction the quantity previously submitted by bidder i is q_i^0 and the next price is p_1. If the bidder is competitive, then the quantity submitted this phase is $q_i^1 = \min\{S_i(p_1), q_i^0\}$.

We note that this definition does not imply that firms have zero profits, even in the case where marginal costs are constant, since the most efficient firms will obtain positive profits bidding competitively. This definition also does not restrict behavior in the second phase; even when marginal costs are constant, a firm will find in its own interest to bid above cost in the price phase, and we still call it "competitive" if it does so.

Uncompetitive behavior decreases not only the revenue but also the efficiency of the market, as uncompetitive bidders withhold generation capacity on purpose to game the market design.

Lemma 4.1 *Suppose $\bar{Q} = Q^*$, marginal costs are constant and all bidders act competitively in the quantity phase of the auction. Let p^* be the maximal price in the price stage and Δ the price decrement used in the quantity phase. Then all bids in the price phase are in the interval $[p^* - \Delta, p^*]$.*

Proof: Since the quantity phase ended when the price dropped from p^* to $p^* - \Delta$, we know that $\sum_j S_j(p^* - \Delta) \leq \bar{Q} = Q^*$.

Since bidding below cost yields negative profits, any bidder knows that the probability of winning with a bid of $p^* - \Delta$ is one. Thus the optimal bid cannot be less than that. $\square$

Using the contrapositive of this result, we obtain a simple test for competitiveness: if $Q^* = \bar{Q}$ and marginal costs are constant, then bidding below $p^* - \Delta$ like observed in the data is evidence of uncompetitive behavior, and therefore of inefficiency.

If $\bar{Q} > Q^*$, as in the data, an additional step must be made to carry out this idea. If the (expected) aggregate supply function varies abruptly between Q^* and $\bar{Q}$, then equilibrium prices can be arbitrarily low. If we know that the aggregate supply function does not vary much, then a lower bound for the prices can be found, based on the outcome of the quantity phase.

4.2 The price phase

Let us focus on the optimal behavior of a generator in the price phase, conditional on the outcome of the quantity phase. In this section we do not assume competitive behavior by anyone. We do assume that this bidder has constant marginal costs. Let c_i and $\bar{q}_i$ be the marginal cost and capacity of this generator.

Let p^* and q_i^* be the maximum price and quantity that generator i can obtain in a given price phase subgame (q_i^* is the minimum of $\bar{q}_i$ and the second-to-last quantity bid from i).

Let $H(p_i)$ be the probability of i winning, if i bids p_i.[4] Because marginal costs are constant, the expected profits of selling 1 MWh with probability 1/2 and 1/2 MWh with probability 1 are the same, and thus we can write that i maximizes

$$\max_{p_i \leq p^*} q_i^*(p_i - c_i)H(p_i)$$

If H is differentiable and i elects to bid less than p^*, the chosen p_i satisfies the FOC, thus

$$c_i = p_i + H(p_i)/H'(p_i)$$

If i bids p^* and we are still willing to assume that H is differentiable at that point, we obtain $c_i \geq p^* + H(p^*)/H'(p^*)$. More realistically, since an atom is expected at p^*, let $p^* - \epsilon$ be the highest available bid below p^*. The cost of a bidder that bids p^* must satisfy $(p^* - c_i)H(p^*) \geq (p^* - \epsilon - c_i)H(p^* - \epsilon)$, thus

$$c_i \geq p_i - \frac{\epsilon H(p-\epsilon)}{H(p-\epsilon) - H(p)}.$$

Once an estimate for H is found, either an estimate or a lower bound of c_i can be calculated using these formulas. From these estimates, and using the assumption of constant marginal costs, we can obtain estimates of the generators profits as well.

Observe that even though this is a multiunit auction, *in the price phase subgame* the quantity sold by each bidder is set. As such, the determination of equilibrium prices is similar to the case of single item auctions. We thus can follow an approach similar to Guerre, Perrigne, and Vuong (2000) to infer marginal costs from observable prices and estimates of price densities and distributions. Here, we need to estimate H and H' from the data.

[4] H also depends on the i's information about its opponents.

5 Learning

We assume that bidders learn about how to form expectations about the other player's behavior. On each auction, they update their beliefs about how the other players will bid in the next auction. We incorporate it in our model in the following way.

First, bidders form expectations about what the other bidders will play based on observable characteristics of each bidder and on a random component. This random term aims to capture the uncertainty of a player about the rivals bidding strategies in a given auction.

Second, bidders use Baye's rule to update their beliefs about their uncertainty about the rival's bids. More specifically, in an auction, after seeing the data, they update the distribution of the random term which will be used to forecast the opponents' bids in the next auction.

$$P(\delta_{t+1}|p_t) = \frac{P(p_t|\delta_t).P(\delta_t)}{P(p_t)}$$

Two things are of interest here. First, we want to introduce further uncertainty on the initial auctions. This is based on the larger dispersion of price bids on the first auctions. If on the other hand we assumed that the auctions were identical, we would be estimating much larger mark ups for these initial auctions, while the annedotical evidence about them points in the opposite direction. Second, we want to quantify the how the market uncertainty evolved and to check if beliefs converged at some point.

6 Estimation of the Model

6.1 Benchmark: Estimating H with homogeneous bidders

While in principle H could be estimated non-parametrically, due to small sample sizes we seek a parametric formulation that is convenient to work with (in particular, it is desirable that the derivative of $\hat{H}$ exists and is well behaved).

In this section, we assume for simplicity that each bidder knows the q_j's of all players right before the price phase starts. Since the c_j's are not public information, the p_j's of the other players are uncertain.

We also make the assumption that p_j is independent of q_j.[5] While this is a strong assumption, it is partially justified by the empirical finding that prices are uncorrelated with quantities in the data (see table 2 above).

Let us assume that i believes its rivals will bid in a way that p_j is censored normal, with mean μ, variance σ^2 and censoring threshold p^*. μ and σ^2 can be estimated directly from price data alone for each auction.

Consider now the problem of a small generator (with q_i that is negligible compared to the overall size of the market). If this generator places a bid t

[5]Note that this is not a consequence of independence in the cost structure — ie., between c_j and $\bar{q}_j$ —, since both p_j and q_j is determined by the realization of c_j.

($\leq p^*$), then the probability of winning is

$$\begin{aligned} H(t) &= \Pr(\sum_{j:p_j \leq t} q_j \leq Q^*) \\ &= \Pr(\sum_j z_j q_j \leq Q^*) \end{aligned}$$

where z_j is a indicator of whether $p_j < t$. Observe that z_j is independent of q_j and is i.i.d. Bernoulli, with $\Pr(z_j = 1) = \Phi((t-\mu)^2/\sigma^2)$, where Φ is the standard normal distribution.

$\sum_j z_j q_j$ is a weighted version of a binomial random variable. Let $\mathcal{J}$ be all subsets of indices $\{1, \ldots, N\}$ such that $J \in \mathcal{J}$ iff $\sum_{j \in J} q_j \leq Q^*$ (in words, quantities with index in J sum up to less than Q^*). Then,

$$\begin{aligned} H(t) &= \Pr(\sum_j z_j q_j \leq Q^*) \\ &= \sum_{J \in \mathcal{J}} \Phi^{\#J} (1-\Phi)^{\#J^c}, \end{aligned}$$

where $\Phi = \Phi((t-\mu)^2/\sigma^2)$.

If N is large, it might not be computationally straightforward to work with $\mathcal{J}$. For example, the problem of maximizing a function over $\mathcal{J}$ is the Knapsack Problem, a well-known example of a NP-hard problem in computer science. For N small, H can be computed by brute force; in this application we have $N = 19$.[6]

6.2 Estimating H with heterogeneous bidders

In this section we drop the assumption that bidders are identical. A quick look at table 1 reveals that homogeneous players is not a plausible assumption in this market. In order to incorporate bidder heterogeneity into the model we keep the assumption that prices are normal distributed but now this distribution depends on observable characteristics of the bidders. To keep things simple we assume characteristics linearly affect the bids distribution mean; $p_j \sim N(X_j\beta, \sigma^2)$. With these assumptions, the problem of estimating the conditional distribution of bids reduces to estimating a standard OLS regression.

The probability of player j bidding p_j greater than all other bids is,

$$\Pr(p_j > p_i) = \prod_{i \neq j} \Phi_i(p_j)$$

Again, $\mathcal{J}$ is the set of all subsets of indices $\{1, ..., N\}$ such that $J \in \mathcal{J}$ iff $\sum_{j \in J} q_j \leq Q^*$. But now each bidder has a distinct distribution, that is common knowledge to all players. Hence, the probability of bidder j selling when bidding p_j is

$$H(p_j) = \sum_{J \in \mathcal{J}} [\prod_{i \in J} \Phi_i(p_j) \prod_{i \in J^c} (1 - \Phi_i(p_j))]$$

[6] It takes about 20 minutes to estimate H for one auction using Matlab 7.5.

6.3 Estimating H: the long list case - heterogeneous bidders, non-observable losing bids and learning

In this section we propose a method that is applicable if the losing bids are not observable. This assumption is in accordance to the data used in the empirical application, and it has two important implications. First, the observed bids distribution is censored. And second, we do not observe the first phase quantities of the losing bids, an information we need to calculate the winning probability.

We take the censoring of bids into account by estimating a censored bid distribution, using a censored regression technique. We estimate a Tobit model where the censoring threshold is the highest winning bid $\overline{p}$. Our model takes the following form:

$$p_i^* = X_i\beta + \varepsilon_i, \quad \varepsilon_i \sim N(0,1)$$
$$p_i = \min\{p_i^*, \overline{p}\}$$

Players forecast player's j bid by using the predicted value from the censored regression plus the stochastic term that represents the uncertainty about bidders' strategies:

$$\hat{p}_j = X_j\beta + \delta_j, \quad \delta_j \sim N(\mu_j, \sigma_j^2)$$

Bidders learn about the stochastic term as they observe the auctions outcomes. To implement this idea in a parsimonious way, we assume that before the realization of auction t player i have a (conjugated) normal prior given by $N(\mu_{i0}, \sigma_{i0}^2)$. Note that each bidder may have a distinct prior. Since we are assuming the price distribution is also normal, the bayesian update of the distribution of the stochastic term has a closed form solution. The mean of the posterior distribution is given by

$$\mu_{it+1} = \lambda\mu_{i0} + (1-\lambda).E(\hat{\varepsilon}_t),$$

where

$$\lambda = \frac{\frac{1}{\sigma_{i0}^2}}{\frac{1}{\sigma_{i0}^2} + \frac{n}{\sigma_{it}^2}}.$$

And the variance is

$$\sigma_{it+1}^2 = (\frac{1}{\sigma_{i0}^2} + \frac{n}{\sigma_{it}^2})^{-1}.$$

The posterior distribution resulting from observing the data of a given auction will be the prior distribution of the next auction.

The quantities are used by the bidders to compute the probability of winning by inferring how much the other players sell at a given bid. Before, we made the simplifying assumption that the quantities defined in the first phase of the auction were public information and players used them together with the probability distribution of bids to calculate their chance of selling the goods. Unfortunately, this is not the case. The other players' quantities sold in the first phase are not observed by the bidders until the auction is over, when the winning bids become public.

In order to overcome this problem, we assume that bidders form expectations about the other bidders quantity bids by running an OLS regression of quantities on bidder characteristics, namely the remaining capacity available for sale, and if the bidder is a federal enterprise. To be more specific, for each auction we run a stacked OLS regression where the dependent variable is the quantity bid on that and all past auctions, and the explanatory variables are the capacity bidders had available for sale in each auction up to that time and if they are a federal enterprise. We also included dummies for the different auctions. We use the estimated coefficients and the values of the characteristics to find the predicted values, which we use as the expected quantities bidders will bid in the auction. This information is assumed to be common knowledge to all bidders.

Summarizing, we estimate the distribution of price bids by using a Tobit model due to the censoring problem of the losing bids. We assume that bidders and an random term to the predicted price (form the Tobit model), and that players use baysian update to learn about the stochastic term. We take into account the fact that bidders do not observe the quantities of the other bidders by assuming that bidders run OLS regressions of quantities on bidders characteristics and use the predicted values as the expected quantities bidders will bid in the auction. With the expected quantities and distribution of bids in hand, we calculated the probability of winning the auction with bid p, $H(p)$. We use the expected quantities to compute all possible elements of $\mathcal{J}$. Each element in this set is a subset of bidders such that the sum of their quantities is less than Q^*.

6.4 Can we estimate $\bar{q}$?

Another question of practical interest is how inefficient is the outcome of the auction (meaning, by how much generators understate their capacities). From our previous discussion about competitive behavior, we have found ways to identify that inefficiencies exist; also, because the quantity-before-price auction is not a Vickrey auction, we do not expect from theoretical grounds its outcome to be fully efficient. It would be desirable to obtain a quantitative assessment of this inefficiency.

One could imagine that, just as costs could be backed out from prices in the pricing phase, capacities could be backed out from quantities in the quantity phase. This is incorrect.

Lemma 6.1 *Consider an instance of the quantity-before-price auction (i.e., specific values for costs and capacities for each generator) and an equilibrium outcome of this game. Take a generator i and let a be the highest proposed quantity by this bidder. For any capacity level $\bar{q}_i > a$, the same outcome is an equilibrium of the game where i has this capacity.*

Proof: $\bar{q}_i$ does not affect i's profits, except as an upper bound on q_i. $\square$

The model does not provide any information on the capacity levels $\bar{q}_i$, beyond the trivial one that they must be above all capacity bids. If one is willing to rely strongly on the assumption that marginal costs are constant up to capacity, this is enough to obtain bounds for the amount of inefficiency in the auction.

7 Data and Estimation Results

Our goal is to estimate the marginal costs of the bids. We use data from the sales of electricity in the first six auctions held by the market regulator. We can only observe the winning bids, both prices and quantities, and the identity and capacity of the bidders.

We used the generation capacity of the bidders in two ways. First, we assumed the total generation capacity differentiate the bidders according to size: a large capacity means a large bidder and so on. Second, we use the capacity to calculate the amount generators had to sell in each auction. Since the contracts overlap, the energy sold in the first auction can not be sold in the second or any of the following auctions. So, the amount a generator has to sell in auction T is given by his total generation capacity minus the amount sold in the prior auctions:

$$\bar{q}_T = \bar{q}_0 - \sum_{t=0}^{T-1} q_t \tag{1}$$

A consequence is that the amount of electricity available for sale decreased across the auctions. And since contracts overlap, so does the total demand.

We are making the assumption that generators had zero energy contracted before the first auction. This was the time when the contracts signed at the time of the reforms of the 1990's were ending, so the anecdote among industry participants was that generators were 'uncovered' by contracts. While it is very likely that they were not completely uncovered, we do not have data about such contracts.

We assume the total demand of the auction Q^* to be equal to the total amount sold in the auction. According to the rules of the auction, the auctioneer had some discretion over the total demand within the auction to increase the competition in case it needed to. So, the total amount sold and Q^* need not to be the same. Since we do not know Q^*, we assume that it is equal to the total amount sold.

7.1 Auxiliary Regressions and Preliminary Results

Table 3 shows the results for the estimation of the censored price distributions. These regressions are used to compute the conditional price distributions that are used as an input in the structural model, but they also bring some insights about the auction results. We regressed prices on the capacity plants had available on the specific auction, as defined in equation 1, and a dummy that indicates if the firm is a federal enterprise. The censoring price p^* is the closing price of the quantity phase (that is, the maximum price in the price phase).

In the first three auctions firms with both large capacity and the federal enterprises bid more aggressively. It seems to be true in the first three auctions but not on the others.[7]

Table 4 shows the results of similar regressions, but in this case we used the percentage (instead of level) of non contracted capacity.

Tables 5 and 6 show the results when we estimate a Tobit model using the data from all auctions together. In this case we included dummy variables for the

[7] We normalized the data for all auctions. For each price i on auction t the normalization was $-(p_i^t - p_{max}^t)$.

Table 3: Price Distribution - Tobit regressions: total of non-contracted capacity

auction 1 (2005-8)			auction 2 (2006-8)		
p1	Coef.	Std. Err.	p2	Coef.	Std. Err.
$\bar{q}_1$	-0.001	0.001	$\bar{q}_2$	-0.001	0.001
fed enterpr.	-2.405	4.664	fed enterpr.	-1.988	3.225
c	1.616	2.196	c	1.755	1.871
auction 3 (2007-8)			auction 4 (2008-8)		
p3	Coef.	Std. Err.	p4	Coef.	Std. Err.
$\bar{q}_3$	-0.003	0.002	$\bar{q}_4$	0.001	0.001
fed enterpr.	-0.889	3.847	fed enterpr.	0.399	2.413
c	6.056	2.996	c	1.369	1.423
auction 5 (2009-8)			auction 6 (2007-8)		
p5	Coef.	Std. Err.	p6	Coef.	Std. Err.
$\bar{q}_5$	-0.0002	0.001	$\bar{q}_6$	0.0001	0.001
fed enterpr.	0.0437	1.849	fed enterpr.	-1.839	2.746
c	0.9661	1.114	c	3.294	2.248

Table 4: Price Distribution - Tobit regressions: % of non-contracted capacity

auction 1 (2005-8)			auction 2 (2006-8)		
p1	Coef.	Std. Err.	p2	Coef.	Std. Err.
			$\bar{q}_2$	-13.96	4.79
fed enterpr.	4.33	3.40	fed enterpr.	2.51	2.26
c	-1.06	1.91	c	10.97	3.76
auction 3 (2007-8)			auction 4 (2008-8)		
p3	Coef.	Std. Err.	p4	Coef.	Std. Err.
$\bar{q}_3$	-1.36	5.86	$\bar{q}_4$	2.66	2.91
fed enterpr.	4.69	4.22	fed enterpr.	-0.53	2.24
c	-3.23	4.99	c	-3.67	2.77
auction 5 (2009-8)			auction 6 (2007-8)		
p5	Coef.	Std. Err.	p6	Coef.	Std. Err.
$\bar{q}_5$	1.00	2.29	$\bar{q}_6$	2.63	3.03
fed enterpr.	0.19	1.85	fed enterpr.	1.97	2.40
c	-1.39	1.66	c	-4.49	2.75

Table 5: Price Distribution - all auctions

	Coef.	Std. Err.
$\bar{q}$	-0,0009	0,0004
fed. enterpr.	-1,115	1,404
D2	-0,429	1,667
D3	1,266	1,776
D4	2,808	1,855
D5	1,457	1,777
D6	3,488	1,973
c	1,643	1,442

Table 6: Price Distribution - all auctions

	Coef.	Std. Err.
$\%\bar{q}$	1.00	2.02
fed. enterpr	-2.40	1.26
D2	0.35	1.73
D3	3.66	1.93
D4	3.39	1.86
D5	3.88	2.02
D6	5.44	2.23
c	1.09	2.39

auctions; D2 is a dummy for the second auction and so on. The omitted category is the first auction. From the results we can see that the price distribution shifted upward as we moved to the later auctions. We can also see that the amount a generator has available to sell $\bar{q}$ negatively affects the its price. It is an interesting result since it could be the case that large bidders (large with respect to the amount of energy one has available to sell, $\bar{q}$) could have more market power than the smaller bidders, and consequently bid a higher price. The data shows the opposite; this results supports the assumption that large players were more eager to contract their energy, and therefore offered lower prices. Also, generators that belong to the federal government bid lower prices than non federal state companies, although this effect is not statistically significant.

Table 7 shows the results of the OLS regression of quantities on covariates. These are the regressions we assume bidders use to forecast how much the other bidders will bid in the auction. One interesting finding is that the capacity available for sale affects the quantity sold in a nonlinear way.

Table 8 shows the evolution of the quantities the plants had available for sale in each auction. A quick inspection on the table reveals that the three federal enterprises, Chesf Eletronorte and Furnas, sold large amounts of energy on the first three auctions, specially Chesf and Furnas. The consequence is that they had little to sell on the next auctions. Since these are large firms, it may explain the fact that large firms bid larger quantities at lower prices.

Table 7: OLS regression of quantities

	model 1		model 2	
	Coef.	Std.Err.	Coef.	Std. Err.
$\bar{q}$	-0.13	0.09	0.24	0.05
$\bar{q}^2$	0.00008	0.00002		
fed. enterp.	117.22	128.62	152.38	151.99
D2	94.19	167.23	-113.44	189.83
D3	-276.61	187.44	-541.06	210.03
D4	-241.64	177.50	-444.39	202.84
D5	-223.15	180.14	-375.63	209.24
D6	-354.56	231.00	-550.40	268.30
c	287.47	145.92	249.83	172.46
R2	0.74		0.62	

Table 8: Quantities Available for Sale - per seller per auction

	$\bar{q}_1$	$\bar{q}_2$	$\bar{q}_3$	$\bar{q}_4$	$\bar{q}_5$	$\bar{q}_6$
CDSA	415	415	415	415	282	282
CEB	130	130	130	130	118	106
CEEE	454	194	42	42	33	24
CELG G&T	12	12	12	12	12	12
CELPA	282	282	282	282	259	259
CEMIG	3733	3733	2806	2806	2701	2701
CESP	3916	3116	1938	1918	1748	1628
CGTEE	270	270	270	270	166	131
CHESF	6254	3754	2700	2562	2112	2032
COPEL	1953	973	605	524	444	199
DUKE	1034	820	762	544	478	478
ELETRONORTE	4164	3492	3164	2614	2524	2524
EMAE	463	378	345	340	337	334
ENERSUL	20	20	20	20	0	0
ESCELSA	557	470	443	443	443	443
FURNAS	6040	2964	437	287	287	6
LIGHT	637	257	127	127	115	115
TEC	340	340	340	340	190	0
TRACTEBEL	2766	2766	2766	2756	2756	2565

Table 9: Auctions 1, 2 and 3 - price, mark up and cost

	auction 1 (2005-8)			auction 2 (2006-8)			auction 3 (2007-8)		
firm	price	-H/dH	c	price	-H/dH	c	price	-H/dH	c
CDSA	0.0	0.48	61.6	0.0	0.49	69.5	0.0	0.12	77.6
CEB	0.0	0.48	61.6	0.0	0.49	69.5	0.0	0.12	77.6
CEEE	57.5	3.54	53.9	67.9	1.45	66.4	0.0	0.12	77.6
CELG G&T	0.0	0.48	61.6	0.0	0.49	69.5	0.0	0.12	77.6
CELPA	0.0	0.48	61.6	0.0	0.49	69.5	0.0	0.12	77.6
CEMIG	0.0	0.48	61.6	69.6	0.62	69.0	0.0	0.12	77.6
CESP	62.1	0.48	61.6	68.4	1.20	67.2	77.7	0.12	77.6
CGTEE	0.0	0.48	61.6	0.0	0.49	69.5	0.0	0.12	77.6
CHESF	52.8	11.97	40.8	60.4	17.66	42.7	66.1	9.96	56.1
COPEL GER	57.5	3.51	54.0	67.6	1.59	66.0	75.4	1.39	74.0
DUKE	60.0	1.53	58.4	70.0	0.49	69.5	76.0	1.08	74.9
ELETRONORTE	56.0	5.26	50.7	63.9	5.19	58.7	77.0	0.51	76.5
EMAE	60.8	1.04	59.8	69.2	0.79	68.4	75.8	1.21	74.5
ENERSUL	0.0	0.48	61.6	0.0	0.49	69.5	0.0	0.12	77.6
ESCELSA	57.0	4.04	53.0	64.0	5.03	59.0	0.0	0.12	77.6
FURNAS	60.9	0.98	60.0	69.6	0.62	69.0	77.7	0.12	77.6
LIGHT	51.7	16.40	35.3	61.1	13.21	47.9	0.0	0.12	77.6
TEC	0.0	0.48	61.6	0.0	0.49	69.5	0.0	0.12	77.6
TRACTEBEL	0.0	0.48	61.6	0.0	0.49	69.5	70.9	4.61	66.3
average		4.9	52.8		4.0	62.7		2.7	71.4
mark up (%)			9%			6%			4%

*This is the ratio of the average mark up and the average marginal cost of the winning bids only.

7.2 Structural Estimates

Tables 9 and 10 show the estimates of the structural model. For each auction, the price, the mark up $(-H/dH)$ and the marginal cost are shown. A zero on the price column means that it is a losing bid. For these cases, the marginal cost shown represents a lower bound for the true marginal cost.

We assume the prior distribution to be $N(0, 100)$. Table 11 shows the evolution of the mean and the variance across the auctions. Note that both the means and the variance converge quickly to a distribution that is similar across the later auctions.

On table 12 we report the results of the model without learning. The results are similar, with the main difference being a decrease of the mark up of the lowest bids. In the model without learning, a lower bid implies a significant lower marginal cost. With uncertainty and learning, a lower bid is credit to a lower marginal cost but also to higher uncertainty about the other players bid. This effect is more pronounced on the first three auctions. The estimated bound for the marginal cost of the losing bids barely change with or without learning.

8 Conclusion

In this paper we study data from the the first six existing energy auctions in Brazil, using a structural model of equilibrium bidding in the price phase of this game.

Table 10: Auctions 4, 5 and 6 - price, mark up and cost

	auction 4 (2008-8)			auction 5 (2009-8)			auction 6 (2007-8)		
firm	price	-H/dH	c	price	-H/dH	c	price	-H/dH	c
CDSA	83.5	0.13	83.4	0.0	0.66	95.3	0.0	0.39	104.6
CEB	0.0	0.11	83.4	94.5	0.97	93.5	0.0	0.39	104.6
CEEE	0.0	0.11	83.4	94.3	1.11	93.2	104.5	0.96	103.5
CELG G&T	0.0	0.11	83.4	0.0	0.66	95.3	100.0	61.02	39.0
CELPA	83.5	0.11	83.4	0.0	0.66	95.3	0.0	0.39	104.6
CEMIG	83.5	0.11	83.4	0.0	0.66	95.3	0.0	0.39	104.6
CESP	83.5	0.11	83.4	93.4	1.83	91.6	0.0	0.39	104.6
CGTEE	83.5	0.11	83.4	91.8	5.16	86.6	0.0	0.39	104.6
CHESF	83.5	0.11	83.4	96.0	0.66	95.3	105.0	0.39	104.6
COPEL GER	82.3	1.14	81.2	96.0	0.49	95.5	0.0	0.39	104.6
DUKE	0.0	0.11	83.4	0.0	0.66	95.3	0.0	0.39	104.6
ELETRONORTE	83.5	0.14	83.3	0.0	0.66	95.3	105.0	0.39	104.6
EMAE	0.0	0.11	83.4	96.0	0.66	95.3	0.0	0.39	104.6
ENERSUL	78.5	12.87	65.6	0.0	0.66	95.3	0.0	0.39	104.6
ESCELSA	0.0	0.11	83.4	0.0	0.66	95.3	0.0	0.39	104.6
FURNAS	0.0	0.11	83.4	96.0	0.66	95.3	0.0	0.39	104.6
LIGHT	0.0	0.11	83.4	0.0	0.66	95.3	0.0	0.39	104.6
TEC	81.6	1.94	79.6	95.0	0.69	94.3	0.0	0.39	104.6
TRACTEBEL	0.0	0.11	83.4	93.0	2.34	90.7	0.0	0.39	104.6
average		2.7	79.4		1.8	92.3		20.8	82.4
mark up (%)			3%			1.9%			25.2%

*This is the ratio of the average mark up and the average marginal cost of the winning bids only.

Table 11: Mean and Variance Evolution - prior and posteriors

firm	1	2	3	4	5	6
mean						
CDSA	0.00	1.36	1.32	2.42	2.02	1.61
CEB	0.00	1.52	1.57	2.89	2.17	2.27
CEEE	0.00	5.90	4.57	5.01	3.21	3.01
CELG G&T	0.00	1.59	1.68	3.08	2.23	1.76
CELPA	0.00	1.44	1.44	2.64	2.08	1.65
CEMIG	0.00	-0.46	-1.37	-1.90	0.50	0.48
CESP	0.00	-0.56	-0.30	-0.26	1.08	1.86
CGTEE	0.00	-0.93	-0.70	0.92	1.42	2.80
CHESF	0.00	4.95	5.10	6.08	4.63	3.15
COPEL GER	0.00	5.05	3.86	4.67	3.76	2.71
DUKE	0.00	3.11	1.77	2.92	2.30	1.77
ELETRONORTE	0.00	2.94	2.36	0.19	1.71	1.27
EMAE	0.00	2.58	2.30	3.79	2.68	2.02
ENERSUL	0.00	1.58	1.67	3.07	4.72	3.34
ESCELSA	0.00	6.30	6.84	6.12	3.87	2.77
FURNAS	0.00	-2.95	-3.06	-0.85	0.53	0.68
LIGHT	0.00	11.45	10.79	9.12	5.29	3.69
TEC	0.00	1.41	1.39	2.55	3.02	2.63
TRACTEBEL	0.00	0.07	-0.76	0.77	1.83	2.41
variance	100.00	1.59	0.65	0.44	0.22	0.14

Table 12: Mark ups (-H/dH) - without learning

	1	2	3	4	5	6
CDSA	0.48	0.49	0.12	0.13	0.75	0.40
CEB	0.48	0.49	0.12	0.12	0.89	0.40
CEEE	3.67	1.44	0.12	0.12	1.00	0.97
CELG G&T	0.48	0.49	0.12	0.12	0.75	63.01
CELPA	0.48	0.49	0.12	0.12	0.75	0.40
CEMIG	0.48	0.62	0.12	0.12	0.75	0.40
CESP	0.48	1.19	0.12	0.12	1.55	0.40
CGTEE	0.48	0.49	0.12	0.12	3.79	0.40
CHESF	13.49	17.45	9.76	0.12	0.75	0.40
COPEL GER	3.63	1.58	1.39	1.11	0.52	0.40
DUKE	1.55	0.49	1.08	0.12	0.75	0.40
ELETRONORTE	5.52	5.13	0.51	0.14	0.75	0.40
EMAE	1.04	0.78	1.21	0.12	0.75	0.40
ENERSUL	0.48	0.49	0.12	12.39	0.75	0.40
ESCELSA	4.20	4.97	0.12	0.12	0.75	0.40
FURNAS	0.98	0.62	0.12	0.12	0.75	0.40
LIGHT	19.33	13.04	0.12	0.12	0.75	0.40
TEC	0.48	0.49	0.12	1.88	0.64	0.40
TRACTEBEL	0.48	0.49	4.56	0.12	1.92	0.40
average	4.9	4.0	2.7	2.6	1.5	21.5
mark up (%)	9%	6%	4%	3%	2%	26%

We find that in the earlier auctions not only the auction price was lower, but the markups were generally higher - the last auction is an exception, due to one very low bid. As in an auction game markups are driven by informational rents, this pattern may indicate that private information among generators dissipated in the latter auctions.

In the context of the methodology proposed here, rising auction prices and diminishing mark-ups lead to the conclusion that marginal costs were rising even more sharply. This may seem odd if marginal costs are determined by the technology of the firms. That, however, is not our interpretation. In our view, the marginal cost for these suppliers reflect their expectations about future market conditions, more specifically how much they expect to sell in the free market 1 MWh on average in the 8 years that correspond to contract being traded.

While data on bilateral contracts in the free market is not available, we can use the spot price as an approximation. From figure 2 we can see that the spot price has indeed risen sharply after the auctions. Thus, the rise of estimated marginal costs can be understood as rational expectations on the part of the generators.

We assume learning into the model. Bidders use baysean learning to update their beliefs about uncertainty concerning the other players. Despite assuming a large variance for the prior, the results do not change substantially from the model without learning. The only difference comes from the lowest bids: when we add prior uncertainty into the model the large mark ups reduce by approximately 15%. We can conclude that without prior uncertainty the higher mark ups would be overestimated. Part of the reason for lower bids was uncertainty about the competitive pressure, and not only market power.

As for the question of whether the federal state companies bid lower prices, we can not be conclusive. The OLS regressions from table 2 show statistical evidence that the federal government owned companies bid lower prices in the first three auctions, but changed their strategy thereafter. A possible explanation is that they contracted most of their energy on the first auctions and had no non-contracted capacity left on the remaining auctions. However, the results from the Tobit regressions, where we control for the capacity firms had available for sale, are not significant even on the first three auctions.

If we look at the results from the structural model we can see that CHESF had marginal costs much lower than the other firms on the first three auctions. It is possible to draw a similar conclusion, although less dramatic, for ELETRONORTE, ESCELSA and LIGHT. These last two companies are not federal state owned companies, so we cannot generalize this result for all federal companies.

One question of interest is to know if firms with large uncontracted capacity bid higher or lower prices. If we look at the results from tables 3 or 5 we can see that the sign of the estimated coefficient ($\bar{q}$) changes across the auctions. This result is consistent with the hypothesis that what matters is the total amount of non-contracted capacity, and not the capacity of an individual plant. The structural model capture this feature well since the larger its total capacity relative to demand, the higher is the probability that a plant does not sell.

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Figure 1: Supply and Demand

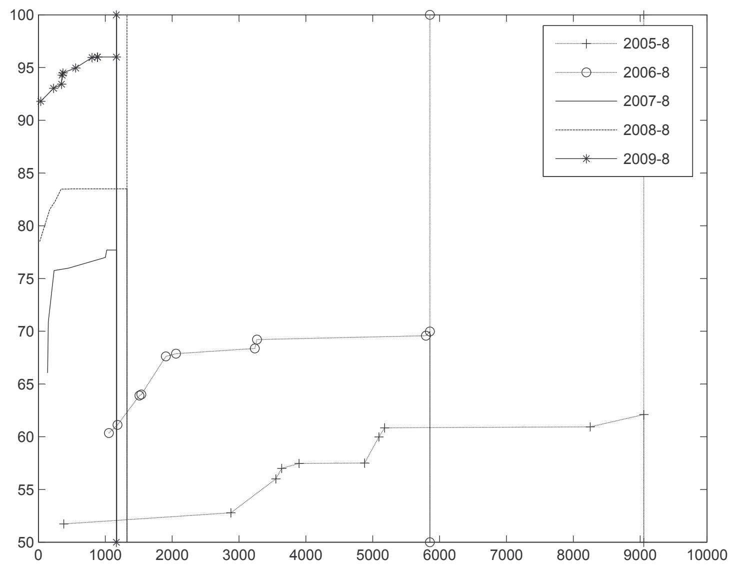

Figure 2: Spot and Auction Prices