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Intervention Function for Smoothing Exchange Rate Volatility: what are the stock and flow implications?

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Intervention Function for Smoothing Exchange Rate Volatility: what are the stock and flow implications?

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We present a hybrid macro-microstructure model in which foreign exchange rate intervention flows are assumed to be analogous to the market order flow (net of buyer-initiated over seller-initiated dealers' trades). Simulations show that small daily drifts in the order flow can lead to major impacts on the stock of foreign reserves. The likelihood of this increases with the Central Bank's inability to adopt a neutral policy with respect to these drifts.
Key Words: exchange rate, interventions, reserves.

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1 Introduction

Many Central Banks (CBs) report to the International Monetary Fund (IMF) that they follow a flexible exchange rate regime. However, quite a few also declare that they perform foreign exchange market interventions. Their stated primary objective is to smooth exchange rate **volatility** with no aim to affect the (trend) level of the exchange rate. The current paper formally investigates the *de facto* flow and stock implications of a policy that closely follows this *de jure* rhetoric[2]. Our main research question is whether a policy function aimed at (smoothing) the second moment of the exchange rate change is neutral with regards to the first moment of the distribution. We also ask whether there are any significant collateral implications for the balance sheets of CBs of such a reaction (intervention) function.

In summary, our findings show that neutrality is observed only if the exchange rate behaves as a driftless random walk or if the CB is perfectly informed about the drift term and adopts a neutral policy. Small drifts in the daily nominal exchange rate can exert non-negligible impacts on the moments of the distributions of both currency returns and the stocks of foreign assets. The likelihood of large variations in their mean levels rises substantially over a matter of a few months in the presence of these drifts.

As can be inferred from standard open macroeconomic models, an interventionist policy that does not aim at impacting the level of the exchange rate would result in a zero net accumulation of foreign assets. However, there are many examples of trends in the stocks of foreign exchange reserves held by CBs across the world. There were many known speculative attacks on emerging economies in the 1990s and 2000s leading to sharp decreases in reserves. They mostly culminated with currency pegs being abandoned in favour of more flexible exchange rates. In the 2000s, these trends were mostly positive in emerging markets [Obstfeld et al. (2010), Menkhoff (2013)], including countries who report to the IMF as floating regimes[3].

As will be discussed, our model can be applied to many of these episodes. However, our empirical analysis will be focused on the case of the Brazilian Central Bank (BCB). This is interesting because, although self-reporting to the IMF as a "floating system", the BCB accumulated around 150 billion USD (United State dollars) of foreign reserves through spot

[2] The rhetoric of intervention for controlling volatility is not at odds with the classification provided by the IMF for floating countries. CBs that intervene exceptionally with an aim "to address disorderly market conditions" provided that "intervention has been limited to at most three instances in the previous six months, each lasting no more than three business days" are classified as free floaters by the IMF [IMF (2017)]. According to the IMF (2017), floating regimes are those in which "...intervention may be either direct or indirect, and such intervention serves to moderate the rate of change and prevent undue fluctuations in the exchange rate, but policies targeting a specific level of the exchange rate are incompatible with floating.". The latest report [IMF (2017)] shows that 31 out of 171 countries are classified as free floaters and 38 as floaters. There are 67 economies which follow an exchange regime with a very small degree of flexibility, i.e. countries with no separate legal tender, currency boards and conventional pegs. Hence, it is likely that almost every country in the IMF sample promote some kind of intervention in the foreign exchange market.

[3] The literature on speculative attacks in emerging economies during the aforementioned period is large. Models are typically concerned with the impact of deteriorating fundamentals and market expectations on the occurrence and timing of these attacks [Krugman (1996)]. See Amador et al. (2016) for a recent speculative attack model which is designed to understand the timing of a **reverse** episode. Their model shows that the desire to peg its currency led the Swiss National Bank to accumulate large positions in foreign reserves in 2015. Concerns about future losses resulted in the subsequent abandonment of the peg.

market operations between 2009 and 2011. This increase in the stock level seems excessive, *prima facie*, if analysed solely from the perspective of the smoothing volatility objective. Estimates show that the net impact of one billion initiated-buys over initiated-sells of USD by the BCB (or, alternatively, the BCB order flow) leads to an approximate 0.5% increase[4] in the exchange rate. Because the BCB's current foreign reserve stock is around 370 billion USD, a 1% offsetting order flow *per* day could, grossly speaking, be generated over a total of 185 days[5]. Hence, the accumulation of foreign assets may imply that there is either a high weight placed on other objectives[6] or there are serious - intended or unintended - collateral effects of the smoothing volatility policy on the stocks of foreign reserves. Our paper investigates the latter hypothesis.

We understand that the goal of smoothing volatility is secondary to the main monetary policy objective: namely inflation control. The instrument used to achieve this primary goal is the interest rate, frequently under the discretion of a monetary policy committee. However, the smoothing problem requires a different instrument or variable, normally under the responsibility of a different sector of the CB. As will be seen, we assume that this variable is the CB order flow. This particular modelling strategy has the advantage of incorporating both the traditional portfolio and signalling channels of the intervention policy literature [see Sarno & Taylor (2001), for example] in an underlying microstructure framework [based on Evans & Lyons (2002)]. In this way, our paper complements not only those numerous works on policy reaction functions for the nominal interest rate but also those that investigate the effects of foreign exchange interventions.

Our model is based on a loss function that places a positive weight on the amount of currency risk faced by economic agents. Deviations from what would have been the observed "free" or "market" exchange rate volatility with respect to an "optimal" level thus decrease the social welfare[7]. The solution to our proposed optimization problem is characterised by a non-linear reaction function for the CB intervention flows. In a nutshell, reactions will depend on the size of the market order flow with respect to a set of thresholds which are deemed "optimal" by this (assumedly) benevolent authority. The reaction function will imprint a non-linear behaviour on the exchange rate, provided that the variance of the underlying shocks are large enough for the exchange rate to exceed, at times, the level of optimal volatility. This behaviour is corroborated by our econometric findings during the period from 01/01/2009 to 30/09/2016, showing evidence of non-linear dynamics in the BRL(Brazilian Real)/USD rate.

Both our analytical and simulation results reveal that if the underlying data generation

[4] This impact is around half of the magnitude that is estimated for the corresponding market order flow - see Barroso (2014), Kohlscheen & Andrade (2014) and Ventura & Garcia (2012).

[5] Which corresponds to more than 70% of the business days generally available in one year.

[6] The literature that investigates the optimal stock of reserves according to social welfare considerations proposes some explanations. A popular one relates to precautionary savings for consumption smoothing purposes during crises or sudden stops episodes - see Jeanne & Rancière (2011), Calvo et al. (2013) Alfaro & Kanczuk (2018). But there are several others: maintain financial stability [Obstfeld et al. (2010)], decrease the impact of debt roll-over risk during sudden stops [Hur & Kondo (2016)], reap dynamic gains from real depreciations, [Levy-Yeyati et al. (2013) and Korinek & Servén (2016)], among others.

[7] We draw insights from the many studies on monetary theory that use quadratic functions to represent welfare gains or losses. See Duarte (2009) for a comprehensive survey.

process of the exchange rate does not contain a drift, net foreign exchange reserve accumulation will, on average, be zero. This will also be the case if there is indeed a drift and the CB adopts a neutral policy with respect to its impact on the exchange rate level. These results clearly contrast with periods in which there are drifts in the order flow arising from speculative attacks, persistent swings in commodity prices or terms of trade, global financial cycles or Peso problems, for instance. Even small daily drifts can *ex post* imply large appreciating or depreciating trends in exchange rates and/or substantial trends in the average stock level of foreign reserves. Our model results and simulations show that the probability of a large variation in the stock of foreign reserves depends considerably on the degree of the "signalling channel" of the CB's smoothing volatility policy. It also depends on the ability of the CB to learn the drift process under uncertainty.

A policy to smooth volatility can be seen as an attempt by policy makers to "round the corners" of the triangle representing the Mundell trilemma [Klein & Shambaugh (2015)]. Ultimately, the geometrical shape that represents the available policy choices can be linked to the drift parameter in our model. A country can choose to have an open capital account, independent monetary policy and relatively stable exchange rates without increasing the likelihood of a major balance sheet problem. However, this will only be true in the absence of drifts and outliers (rare events). If the CB intentionally or unintentionally buffers those drifts in the level of foreign reserves, a quadrilemma [Aizenman (2011)] could exist. An additional vertex representing financial stability may only be attainable by means of (de)accumulating large levels of international reserves. Our model also rationalises the idea that a global financial cycle, as suggested by Rey (2015), could reduce the trilemma into a dilemma. The reason is that persistently tight or loose monetary policies in the centre economies could export exchange rate drifts to the smaller open economies. In turn, these drifts will significantly increase the probability of massive changes in the level of foreign reserves of these smaller markets, thus raising concerns of macroeconomic instability.

The rest of this paper is structured as follows: the next section briefly reviews the related theoretical and empirical literature. It provides the justification for the model, econometric estimations and calibrations. We introduce our smoothing volatility model in the third section. The fourth presents the data as well as the simulation results and corresponding analysis. The last part of the paper presents our concluding remarks.

2 Portfolio Balance, Signals and Microstructure

The literature that investigates the impact of direct interventions by monetary authorities is very large, so we will concentrate on a few papers. First, it has been noted that interventions are more relevant for emerging economies than for developed ones[8] [Daude et al. (2016)] and that substantial increases in foreign reserves have been observed recently in these countries. Second, the literature broadly suggests the existence of two simultaneous channels by which interventions affect the exchange rate: the portfolio balancing and the signalling channels [see Sarno & Taylor (2001)]. Third, it also recognises the importance of microstructure in

[8]The reasons are various. Menkhoff (2013), for instance, cites a few: the larger relative size of CBs, restrictions for other banks to operate in the market and so on.

understanding the mechanisms through which these channels are linked to exchange rates.

There are various views on the workings of the signalling channel. For example, interventions can provide a signal of the future stance of monetary policy [Mussa (1981)] and convey information about the future values of macroeconomic variables [Hung (1997)]. It is clear that the strength of this channel depends on the credibility of the monetary authority [Dominguez (1990), Sarno & Taylor (2001)]. There is also evidence that these channels can have an impact on volatility [Reitz & Taylor (2008), Vitale (2008), etc].

The portfolio balancing channel depends on the hypothesis that local and foreign assets are imperfect substitutes and that agents are risk averse, so that any sterilized intervention could impact on the exchange rate by affecting the relative supply of assets [Tryon (1983)]. Although it is difficult to disentangle one effect from the other, the empirical importance of these two channels have been found to be important [Dominguez & Frankel (1993)]. Theoretical results and empirical evidence point to mixed results with particular regards to the impact of these two channels on volatility [see Sarno & Taylor (2001)]. There are cases in which exchange rate interventions may have been successful in reducing exchange rate volatility [Dominguez (2006), for example]. However, there are also results showing that, on other occasions, interventions may have intensified exchange rate volatility [Baillie & Osterberg (1997), for instance].

Microstructure models of the exchange rate, which draw on Evans & Lyons (2002), provide an alternative answer to the enigma of international macroeconomics about the apparent disconnect between macroeconomic fundamentals and the short term behaviour of exchange rates [Obstfeld & Rogoff (2000)]. The basic idea is that, at the beginning of a typical trading period, customers observe their particular state of nature, form their expectations and place orders in the currency market. Customer orders result in negotiations in the interbank tier of the market and consequentially, order flow. The latter aggregates all information regarding underlying price pressures which help dealers to learn about the observed macroeconomic state of nature. This, in essence, is the fundamental source of agent heterogeneity [see Bacchetta & van Wincoop (2006) for a model with these main ingredients]. Optimal portfolios are allocated in equilibrium through price changes that reflect optimal risk sharing between a small number of dealers and a great number of customers. In other words, equilibrium prices will be such that, at the end of the trading day, customers will absorb the unexpected order flow. Hence, at the core of the microstructure models rests a portfolio rebalancing/readjustment mechanism between the interbank/customer market [Evans & Lyons (2002) and Evans (2011)]. We understand that a comprehensive model of interventions, aimed at understanding the smoothing volatility problem, should include the two channels described above in a comprehensive microstructure framework.

Although our broad review aims at understanding the intervention channels as well as mapping our research problem and contribution to the related literature, we have another specific focus. We also intend to identify the main findings of the papers that were primarily concerned with the effect on BRL/USD volatility. We believe that the model put forward in the next section can be used - *prima facie* - to help with the understanding of the interventionist policies of other CBs across the world. Our application, however, will be aimed at the particular impact of BCB interventions. There are two main reasons. First, the BCB explicitly states that one of its policy objectives, within a framework of inflation target and floating exchange rates, is to avoid excessive currency volatility[9]. The second is that there is a growing literature on this topic in Brazil, with several estimates showing that interventions were important. In fact, these estimates will later be used for the calibration of our model.

There are two main remarks to be made with respect to the findings regarding both the dynamics of the BRL/USD exchange rate, in general, and the impact of BCB interventions, in particular. First, it has been noted that interventions have been substantial and seem to have had significant impacts on the BRL/USD exchange rate. The bulk of these empirical investigations were performed using conventional econometric techniques such as VARS and other strategies that allow identification of the structural parameters. They mainly recognise the bias that arises from the endogeneity by simultaneity[10]. There are also novel and interesting methodological applications such as the synthetic control approach used by Chamon et al. (2017). The former created a counterfactual exchange rate using data of covariates (currencies) that were not subject to the massive interventionist currency swap program implemented by the BCB between August 2013 and April 2015. Chamon et al. (2017) found that interventions over-appreciated the currency at the beginning of this program by 10% with respect to their counterfactual level. The average effect on volatility was found to be zero using the squared change in the log of the exchange rate, however they present results showing a decrease of around 10% of an option-implied volatility measure[11]. Finally, it must be noted that the estimated effect of interventions has a similar magnitude, albeit slightly smaller, to that of the market's order flow [see Barroso (2014) and Kohlscheen & Andrade (2014)]. The second remark can be made concisely, but is also very important for our paper: there is evidence that the USD/BRL exchange rate behaves is a non-linear fashion [see Simonassi et al. (2012), for example].

3 The Smoothing Volatility Model

The simple model put forward in this section has a practical objective. As previously mentioned, it aims to help CBs in their understanding of the foreign reserve (stock) implications of optimal (flow) policies of smoothing exchange rate volatility. Although our model was designed to understand interventions in countries that follow a *de facto* managed-floating exchange rate regime, our results can be applied to a wider range of classifications. Our empirical application will be for the specific case of a country that has both an inflation target and a floating exchange rate regime. There is evidence for a large sample of countries that interventions designed to smooth volatility are effective for economies with this sort of arrangement - see Berganza & Broto (2012) and Fratzscher et al. (2019), for instance.

[9] It also states that the accumulation of foreign assets also contribute to the sustainability of the Balance of Payments. This explanation is not easily found as a mission statement of the BCB. It can be seen in the series of frequently asked questions, which is only available in portuguese in the following link: *http://www.bcb.gov.br/conteudo/home-ptbr/FAQs/FAQ%2011-Fun%C3%A7%C3%B5es%20do%20Banco%20Central.pdf*.

[10] One example is Barroso (2014) who addresses the endogeneity problem and find that interventions are able to reduce exchange rate volatility.

[11] An impact which was not observed when the program was extended. The ambiguous impact is not a novelty - Oliveira & Plaga (2011) provide an analysis of the impact of interventions on volatility using daily data for the period between 1999 and 2006 and an E-GARCH model, finding mixed results.

Having reviewed the literature in the previous section, we chose to build a theoretical model with an underlying microstructure framework for the foreign exchange market that takes into account both the signalling and portfolio channels. We will consider that the magnitude of the price impact of the CB intervention is analogous to that of a market order flow, i.e. it occurs through the net result of initiated-purchases over initiated-sales of foreign reserves. In other words, the model explicitly assumes that the signalling/portfolio channel, as characterised in the previous section, is given by means of a variable that is conceptually identical to the market order flow[12].

3.1 Loss Function for the Exchange Rate Policy

Consider that there exists a "desk" in the CB which is uniquely charged with the execution of the economy's exchange rate policy. We will however, make reference to this "desk" in general terms, i.e. as the CB itself. Assume also that its main objective, or mission, is to control the amount of foreign exchange rate risk faced by dealers (and, ultimately, by customers)[13]. In the model put forward below, the CB minimises deviations of risk premium from an "optimal" level by choosing the amount of FX intervention.

Our model takes as given the stylised classification of a typical trading day from foreign exchange rate microstructure models, as in Evans & Lyons (2002). This classification considers that the day is divided into three main rounds, the first comprising of transactions between customers (i.e. non financial agents or, interchangeably, the public) and the dealers. The second round mainly consists of interdealer transactions. Finally, the third round is characterised by transactions between dealers and the public at the end of the trading day. Optimal risk sharing is achieved through an exchange rate change which enables a transfer of the dealers' accumulated excess inventory back to the public.

Assume that the benevolent CB has the following social welfare function[14]

[12]These ideas draw insights from both the theoretical and empirical findings of the microstructure literature, specially with regards to the informational content of the order flow. Our reduced form assumption for the exchange rate, which will depend on both the market and the CB order flows, is based on a portfolio adjustment model along the lines of [Evans & Lyons (2002)]. The portfolio balancing effect occurs because the equilibrium price in these models largely reflects optimal demands given by optimal risk sharing considerations. The underlying structure of the model considers that the order flow in the interbank market, generated from customer orders in the primary tier of the market, depends on unexpected changes in agents' private incomes as well as public news. These orders thus provide information or signals fundamental changes in the economy. On aggregate, they transmit this information to prices through order flow - which simultaneously reflects portfolio adjustments.

[13]This is one example of a possible microeconomic justification for the smoothing policy. We can think of other welfare considerations that would lead to similar results. It is worth emphasising that exchange rate volatility can impact on the balance sheet of private business, economic growth, international trade, inflation and inflation expectations etc. A crucial paper that explains why volatility is particularly distressful for emerging economies is Calvo & Reinhart (2002).

[14]Note that this modelling option is actually based on the literature, already quite consolidated, which suggests the existence of a monetary policy reaction function for the level of the nominal interest rate. Traditional approaches, such as in the dynamic inconsistency problem of monetary theory, have a loss function for the benevolent representative of the society which is based on deviations of inflation from optimal as well as income from its socially optimal level [Kydland & Prescott (1977)]. According to Sarno & Taylor (2001)'s review, conventional theoretical analysis focus on designing an exchange rate regime which minimizes the

$$\min_{\{I_t^{(j)}\}_{t=1}^{T}} \mathscr{L}^s = \sum_{t=1}^{T} \left\{ u \left[\boldsymbol{E} \left(W_t^{(j)} \right) \right] - \boldsymbol{E} \left[u \left(W_t^{(j)} \right) \right] - \sigma^2 \right\}^2, \tag{1}$$

where $\boldsymbol{E}$ is the expectation operator, time subscripts refer to the end of (the third round of negotiations of) a typical trading day, $W_t^{(j)}$ is the aggregate wealth held by the dealers, $\sigma^2 \geq 0$ stands for the optimal amount of risk premium, evaluated in utility units, "targeted" by the CB. Some variables carry the superscript $j \in \{-, 0, +, F, N+, N-\}$ because of the non-linear nature of the intervention, which is given by $I_t^{(j)}$. This will be later explained.

Furthermore, consider that σ is exogenously given, utility is concave and, for simplicity, a quadratic functional form[15], $u\left(W_t^{(j)}\right) = aW_t^{(j)} - \frac{1}{2}\left(W_t^{(j)}\right)^2$, defined over the range in which marginal utility is always positive $\left(a > W_t^{(j)}, \forall t\right)$. With these assumptions, and abstracting from the effects of strategic interactions among agents in the FX market, the welfare problem can be written as

$$\min_{\{I_t^{(j)}\}_{t=1}^{T}} \mathscr{L}^s = \sum_{t=1}^{T} \left\{ \boldsymbol{E} \left[\left(W_t^{(j)} \right)^2 \right] - \left[\boldsymbol{E} \left(W_t^{(j)} \right) \right]^2 - \sigma^2 \right\}^2. \tag{2}$$

Given that the variance of wealth is $\text{Var}\left(W_t^{(j)}\right) \equiv \boldsymbol{E}\left[\left(W_t^{(j)}\right)^2\right] - \left[\boldsymbol{E}\left(W_t^{(j)}\right)\right]^2$, the problem then becomes

$$\min_{\{I_t^{(j)}\}_{t=1}^{T}} \mathscr{L}^s = \sum_{t=1}^{T} \left[\text{Var}\left(W_t^{(j)} \right) - \sigma^2 \right]^2. \tag{3}$$

Assume that domestic dealers' wealth, denominated in domestic currency, is given by

$$W_t^{(j)} = \left(1 + \Delta s_t^{(j)}\right) S_{t-1}^{(j)} A_{t-1} + d_t S_{t-1}^{(j)} B_{t-1}, \tag{4}$$

where A_{t-1} is the optimal cash holdings of foreign exchange held by the dealers at the third round of day $t-1$, d_t are the dividends received in foreign currency *per unit* of FX investment, B_{t-1} is the quantity of foreign bonds invested in $t-1$, S_t is the spot exchange rate at t and s_t is the correspondent natural logarithm; Δ stands for the first difference, A_0, S_0 and B_0 are given. Our assumption that t refers to the end of the third round of negotiation on day

variance of output deviations from its natural rate. Glick & Hutchison (1989), a pioneer in the literature of optimal intervention, also include deviations of the domestic price level from a long run equilibrium value in their loss function. See Duarte (2009) for an interesting discussion on the historical use of quadratic functions in post war monetary economics.

[15] Other functional forms could have been alternatively assumed. In the case of a constant relative risk aversion function, for instance, our model story would include not only FX demands derived by risk premium considerations (the impact of the variance of wealth on the expected level of utility) but also for precautionary reasons (the effect of the variance of wealth on the expected marginal utility). For algebraic simplicity, we will focus on the first.

t implies that CB's FX intervention will occur simultaneously with the risk sharing of FX inventory holdings between dealers and the public.

We will take a simplified approach and choose a parsimonious specification for the variance of wealth in (4). As will be seen, we only consider the variance of the exchange rate change in the problem. This is justified on the basis that d_t is exogenous, that optimal cash holdings by dealers tend to be very small (and possibly relatively constant) at the end of trading days and/or because we assume that the variance of the exchange rate change tends to dominate the overall variance of wealth.

3.2 Order Flow Rules

Assume that the problem of the monetary authority can be simplified to the following

$$\min_{\{I_t^{(j)}\}_{t=1}^{T}} \mathscr{L}^s = \sum_{t=1}^{T} \left[\left(\Delta s_t^{(j)} \right)^2 - \sigma^2 \right]^2, \tag{5}$$

where, using insights from Andersen et al. (2003), we consider that $\left(\Delta s_t^{(j)} \right)^2$ is the realised volatility[16]; σ^2 now has a different interpretation, as it stands for the optimal level of volatility[17].

Microstructure models of the exchange rate [Evans & Lyons (2002) and Evans (2011)] suggest that the daily variation of the exchange rate is primarily a linear function of the unexpected order flow, hence our hypothesis is that

$$\begin{aligned} &\Delta s_t^{(j)} = i_t^n + \varepsilon_t^{(j)} \\ &\text{and} \\ &\varepsilon_t^{(j)} = \beta x_t + \theta I_t^{(j)}, \end{aligned} \tag{6}$$

where i_t^n represents public news on the interest rate differential, x_t is the market order flow and $I_t^{(j)}$ corresponds to the "order flow" resulting from the CB's (official, unexpected, unsterilised and direct) intervention rule. The parameters β and θ represent the impact of the market and CB order flows, respectively, on the exchange rate. If the monetary authority is credible and has a greater information set than that of the market, one would expect that $\theta > \beta$. It

[16]As suggested by (5), the monetary authority has a symmetrical loss every time the realised volatility is different from the target. However, the end of the day change carries the j superscript from the non-linear CB reaction function. Hence, there will be restrictions on the first order conditions that characterise the optimal solution of the unconstrained problem. These restrictions on the policy available choices will imply that some variables are non-linear. Accordingly, there is an implicit non-linear welfare assumption. We could have established non-linearity directly by placing symmetric weights (restrictions) into the CB welfare function. This could be done, for example, by assuming a piecewise loss function: $\mathscr{L}^s \times \mathbb{1}(E_t)$, where $\mathbb{1}$ is the indicator function, i.e. for any event realisation (E_t), $(E_t) = 1$ if E_t is true and $(E_t) = 0$ otherwise. For expositional reasons, j will be characterised later.

[17]Although this is assumed to be exogenously defined, it is possible to conjecture e that it depends on domestic and international macroeconomic fundamentals, agents' preferences or risk aversion, among others. Our paper does not, however, propose to investigate its determination.

follows that the difference $(\theta - \beta)$ could be measuring the relevance of the signalling channel. *Leaning against the wind* factors, as suggested by some versions of the Taylor rule [Taylor (1998)], are also indirectly incorporated into the loss function via the i_t^n term in $\Delta s_t^{(j)}$. We assume that the overall market order flow, x_t, is divided into two different parts:

$$x_t = \gamma_t + \eta_t, \tag{7}$$

where η_t is the unexpected order flow in the absence of a drift regime which is, in turn, represented by γ_t. In fact, we will not make *a priori* assumptions about the data generating process of the drift[18]. However, we understand that drifts could exist for several reasons. Swings in commodity prices that affect terms of trade could generate small but persistent daily order flow trends or, for example, Peso problems that affect the level of hedge demands for foreign exchange in some predictable way.

As shown in the previous section, there is evidence pointing towards the existence of temporal structure in the conditional volatility of the exchange rate. We incorporate dynamics by assuming that the market order flow behaves according to

$$\eta_t = v_t \sqrt{h_t}, \tag{8}$$

where $v_t \sim \mathcal{N}(0,1)$ and h_t is assumed to follow a GARCH (p,q) specification, where p and q are the orders of the conditional volatility terms[19].

The CB problem

In summary, the problem of the monetary authority, which will be solved at each (end of) time period t, after having observed the market order flow, can be represented as

$$\begin{aligned}
\min_{\{I_t^{(j)}\}_{t=1}^T} \mathscr{L}^s = \quad & \sum_{t=1}^{T} \left[\left(\Delta s_t^{(j)} \right)^2 - \sigma^2 \right]^2 \\
\text{subject to} \quad & \Delta s_t^{(j)} = i_t^n + \varepsilon_t^{(j)}, \\
& \varepsilon_t^{(j)} = \beta x_t + \theta I_t^{(j)}, \\
& x_t = \gamma_t + \eta_t, \\
& \text{and } \eta_t = v_t \sqrt{h_t}.
\end{aligned} \tag{9}$$

For simplification purposes, we will further ignore the effect of public news on the the daily change in the exchange rate by establishing that $i_t^n = 0, \forall t$. Assuming differentiability, the first order necessary conditions can be shown to imply that

[18] Our overall specification preserves the assumption that the market exchange rate follows a unit root process, with or without a drift. This is in accordance with the conclusions that can be drawn for a daily frequency from the literature that followed from Meese & Rogoff (1983)' seminal paper.

[19] We will later use both ARCH(1) and GARCH(1,1) processes for the simulations - the assumption of one lag is for simplification purposes only. Also observe that the ARCH/GARCH structure is considered for the market order flow instead of the CB's order flow, since the latter is modelled as consequence of the policy response to market conditions.

$$I_t^{\mathrm{F}} = -\frac{\beta x_t}{\theta} \text{ and } \sigma = 0, \tag{10}$$

or $\sigma \neq 0$ and

$$I_t^{(+)} = \frac{-(\beta x_t - \sigma)}{\theta}, \tag{11}$$

or

$$I_t^{(-)} = \frac{-(\beta x_t + \sigma)}{\theta}. \tag{12}$$

Equation (10) represents the trivial way of solving the smoothing volatility problem when the government chooses $\sigma = 0$. The CB then adopts a policy rule for the order flow as in (10), which corresponds to the adoption of a fixed exchange rate regime. This solution will not be considered in our analysis. Equations (11) and (12) give the other two possible solutions when $\sigma \neq 0$ and finite.

Non-linear reaction function

Because of the quadratic nature of the problem, the CB could, in principle, react to an increase in the exchange rate above the desired volatility level by choosing $I_t^{(-)}$ and producing a change of $-\sigma$. It could also adopt a policy rule like $I_t^{(+)}$ to a decrease below target, generating a $+\sigma$ variation.

However, we assume that the CB will preserve the observed direction of the exchange rate change when applying its policy rule. Alternatively, we suppose that after deciding to react according to (10) and (11) the CB will minimise the impact of its intervention on the level of foreign reserves. We will also consider that the monetary authority does not have the objective of generating volatility when it is smaller than σ^2. In other words, CB intervention flows will only happen on occasions in which the volatility is above the desired level. These considerations imply that we can posit (11) and (12) as non-linear solutions, according to the following conditions[20]

[20] As a matter of fact, our choice of notation is complicated by the simultaneous nature of the variables in the optimisation problem. We could have alternatively used, for instance, the concept of a "shadow" intervention function as in Kearns & Rigobon (2005). Suppose that I_t^* represents the shadow intervention in the CB loss function. Assuming $\sigma > 0$, the "actual" intervention function would take the form $I_t = \mathrm{sgn}(x_t) \times I_t^* \times \mathbb{1}(|\beta x_t| > \sigma)$, where $\mathbb{1}$ is the indicator function, as defined earlier, with $E_t = \{\beta x_t > \sigma, \beta x_t < -\sigma\}$. The two roots of the quadratic problem would then be chosen according to $\mathrm{sgn}(x_t)$: $\mathrm{sgn}(x_t) = -1$ if $x_t > 0$ and $\mathrm{sgn}(x_t) = +1$ if $x_t < 0$, given the considerations above.

$$j = \begin{cases} + & \text{if } \beta x_t > +\sigma, \\ - & \text{if } \beta x_t < -\sigma, \\ 0 & \text{if } +\sigma \geq \beta x_t \geq -\sigma. \end{cases}$$

The rules proposed above show that the intervention is contingent upon the realisation of the market order flow and the thresholds $\{-\sigma, +\sigma\}$. This hypothetical behaviour imprints a non-linear dynamics into the exchange rate, which is consistent with the findings of the literature presented earlier [see Reitz & Taylor (2008) and Kohlscheen (2012)] and with the results shown in the next section. In summary, our hypotheses imply that when $\beta x_t > +\sigma$ and $\beta x_t < -\sigma$, respectively, the two reduced form equations (and reflexive barriers for the exchange rate for the cases in which $\sigma \neq 0$) are thus

$$\begin{aligned} \Delta s_t^{(+)} &\equiv \beta x_t + \theta \left[\frac{-(\beta x_t - \sigma)}{\theta} \right], \\ &= +\sigma, \end{aligned} \tag{13}$$

and

$$\begin{aligned} \Delta s_t^{(-)} &\equiv \beta x_t + \theta \left[\frac{-(\beta x_t + \sigma)}{\theta} \right], \\ &= -\sigma. \end{aligned} \tag{14}$$

Results (11) and (12) show a reaction from the CB in the opposite direction of the market order flow which does not eliminate volatility but rather controls or "smooths" it.

Drifts, uncertainty and neutral policies

Our assumption that the drift term is not necessarily constant serves one specific objective. We aim to analytically investigate the impact of the CB uncertainty with regards to the drift on the model results. One straightforward way to characterise the learning process is to suppose a state space representation. A simple way to proceed with the analysis is to consider that γ_t and η_t are both independent zero mean processes and to ignore the conditional variance in η_t. The rational expectation forecast of the CB can thus given by a typical signal extraction problem

$$E(\gamma_t | x_t) = \hat{\gamma}_t = \kappa x_t, \tag{15}$$

where σ_γ^2 and σ_η^2 are the unconditional variances of γ_t and η_t, respectively and $\kappa \equiv \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \sigma_\eta^2}$. Now, suppose that the CB does not wish to react to this drift term. In fact, this is the "de jure" policy of many floating countries. In this case, the rules can be modified to be "neutral" with regards to the effect of the drift term

$$I_t^{(N+)} \equiv \frac{-\beta[x_t - \hat{\gamma}_t] + \sigma}{\theta} = \frac{-\beta[(1-\kappa)x_t] + \sigma}{\theta}, \tag{16}$$

and

$$I_t^{(N-)} \equiv \frac{-\beta[x_t - \hat{\gamma}_t] - \sigma}{\theta} = \frac{-\beta[(1-\kappa)x_t] - \sigma}{\theta}. \tag{17}$$

when $\beta x_t > +\sigma$ and $\beta x_t < -\sigma$, respectively.

In order to analyse these neutral rules, consider first the case in which $\sigma_\eta^2 \to 0$. It follows that all variation in the order flow will be due to changes in the drift term and hence the signal is perfectly informative ($\kappa \to 1$). The drift will almost certainly be known by the CB. The rule then prescribes that the order flow for the threshold cases shown above must be $I_t^{(N+)} = \frac{+\sigma}{\theta}$ and $I_t^{(N-)} = \frac{-\sigma}{\theta}$, respectively. The corresponding exchange rate changes will be, $\Delta s_t^{(N+)} = \beta x_t + \sigma$ and $\Delta s_t^{(N-)} = \beta x_t - \sigma$. The neutral rule and the hypothesis of a perfectly informed CB both imply that the exchange rate will entirely absorb the impact of the drift term. In the case when $\sigma_\eta^2 \to \infty$, however, there is too much noise in the signal and the CB is not able to learn anything about the drift from the observation of the market order flow. The signal will thus be completely uninformative ($\kappa \to 0$). The neutral rules that are associated with a perfectly uninformed CB are thus analogous to the ones in which the CB does not react to the drift term, i.e. (11) and (12).

Empirical Model: in the cases when the CB either ignores and it is informed or when it is perfectly uninformed about the drift term (or simply when there is no drift), the behaviour of the exchange rate can be described in the following way:

$$\Delta s_t^{(j)} \equiv \begin{cases} +\sigma & \text{if} \quad \beta x_t > +\sigma, \\ -\sigma & \text{if} \quad \beta x_t < -\sigma, \\ \beta x_t & \text{if} \quad -\sigma \leq \beta x_t \leq +\sigma. \end{cases} \tag{18}$$

It can be shown that an analogous non-linear process exists for the cases in which the CB is imperfectly informed and does not ignore the drift. The above discussion suggests a general unrestricted non-linear model that includes several lags and constant terms. Dropping the j superscript, we can posit the following Self Exciting Threshold Autoregressive (SETAR) model:

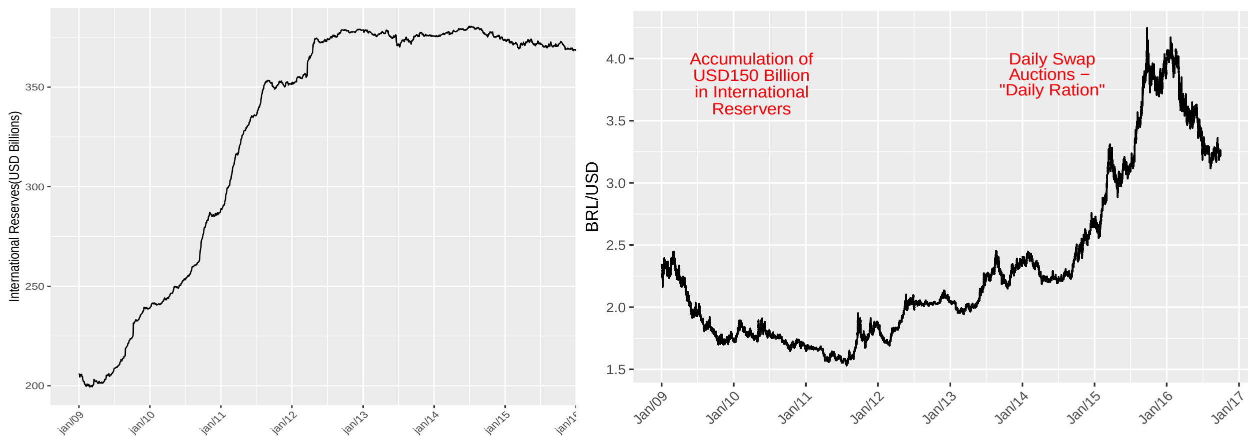
$$\Delta s_t \equiv \begin{cases} \phi_{10} + \phi_{11}\Delta s_{t-1} + \cdots + \phi_{1L}\Delta s_{t-L} + \xi_t & \text{if} \quad \Delta s_t \leq -\sigma, \\ \phi_{20} + \phi_{21}\Delta s_{t-1} + \cdots + \phi_{2L}\Delta s_{t-M} + \xi_t & \text{if} \quad -\sigma < \Delta s_t \leq +\sigma, \\ \phi_{30} + \phi_{31}\Delta s_{t-1} + \cdots + \phi_{3L}\Delta s_{t-H} + \xi_t & \text{if} \quad +\sigma < \Delta s_t, \end{cases} \tag{19}$$

where L, M, H are the maximal lags in the low, intermediary and high regimes, respectively; ξ_t is an $i.i.d \sim (0, \sigma_\xi)$ shock; the thresholds in the reduced form above could be viewed as the reflexive barriers imposed by the CB chosen degree of optimal volatility. Estimations of the model in (19) using observed data for the USD/BRL currency pair will be shown in the next section. We will also perform simulations using (11) and (12) mainly to evaluate the impact of the intervention rules on the stock of assets held by the CBs.

4 The case of the BCB

Our sample data spans from 01/01/2009 to 30/09/2016 and was obtained from the *Bloomberg* terminal and from the BCB time series system. We collected the nominal BRL/USD exchange rate for every 5 minutes interval on a total of 1905 days[21], with 108 daily observations. This period was chosen for two main reasons. Firstly, there was a significant increase in the amount of foreign reserves held by the BCB between January 2009 and July 2011, as can be seen in Figure 1a. Secondly, from August 2013 to March 2015, the BCB implemented a large program of currency swaps auctions with the objective of providing hedge and liquidity to the currency market. This program started with a **daily** supply of 500 million USD and, for this reason, was later named the "daily ration" program.

Figure 1: BCB Reserves and the Exchange Rate



(a) Foreign Reserves

(b) BRL/USD Exchange Rate

Notes: Both figures present daily data between 01/01/2009 and 30/09/2016. Figure 1a shows reserves calculated by the BCB using the liquidity concept. Figure 1b presents the spot exchange rate collected from Bloomberg.

Visual inspection of Figure 1b. suggests a non-stationary behaviour for the exchange rate. Using the Augmented Dickey Fuller (ADF) test, considering a constant and a time trend, one cannot reject the null hypothesis of non-stationarity as shown in Table 1[22]

[21]Days in which there were no transactions, due to holidays, for example, were not considered.

[22]Applying the same type of ADF test but for each day separately, one can verify that it is not possible to reject the hypothesis of non-stationarity in 1665 out of the 1905 days of the sample, using a p-value of 0.1. These results are not reported for the sake of space.

Table 1: Unit Root Test

Augmented Dickey-Fuller	
Series:	log(BRL/USD)
Ha:	Stationary
ADF =	-2,000
P-Value =	0,5791

Notes: Intra-daily data between 01/01/2009 and 30/09/2016. Prices recorded every 5 minutes.

The assumption that the exchange rate and, by deduction, the order flow have a conditional volatility structure was based on the findings of the related literature as well as on the results shown on Table 2. As can be seen, there is evidence to reject the null hypothesis of no conditional volatility (heteroskedasticity) using the Lagrange Multiplier test (LM)[23].

Table 2: Test for the ARCH Structure

Lagrange Multiplier	
χ^2 =	105.280
Lag=	12
p-value =	0,000

Notes: Intra-daily data between 01/01/2009 and 30/09/2016. Prices recorded every 5 minutes.

We also performed a test of non-linear structure in the series. As shown by Table 3 below, the null of linearity is rejected at the 10% significance level for a SETAR(3).

Table 3: Linearity test

Hansen test	
Ha:	SETAR(3)
p-value:	0,1

Note: non-linearity test for the exchange rate between 01/01/2009 and 30/09/2016. See Hansen (1999) for a review of non-linear approaches such as the SETAR model.

[23]If the analysis is performed for each day separately, one can observe the non-rejection of the null for 1542 of the 1905 days of the sample.

4.1 Non-linear reduced-form

Results presented in Table 3 suggest the existence of non-linear dynamics in the series. We then estimated a SETAR model with Δs_t as the dependent variable. The thresholds trigger a regime change with one lag. They were chosen based on a best fit grid search using all possible values in the sample (i.e. the values of the lags of the dependent variable itself), with at least 15% observations in each regime. Table 4 shows the values of the estimated coefficients. As can be seen, the median of the coefficient in the middle regime, represented by ϕ_{21}, is larger in absolute value than the ones in the upper and inferior regimes (ϕ_{11} and ϕ_{31}), respectively. The same result is observed when one considers the 1^{st} and the 3^{rd} quartiles. Results presented in Table 4 reveal that the first difference of the exchange rate series is slow mean reverting in the middle regime (by comparison to the outer regimes) which suggests that our model is likely to be capturing the actual behaviour of the exchange rate variable.

Table 4: SETAR Model Results

	ϕ_{11}	ϕ_{21}	ϕ_{31}
Minimum	-3,500	-16,290	-2,680
P(25)	-0,371	-1,844	-0,427
Median	-0,060	-0,167	-0,109
Average	-0,091	-0,073	-0,121
P(75)	0,219	1,507	0,190
Maximum	2,107	21,63	1,899

Note: Intra-daily BRL/USD data. Prices recorded every 5 minutes.

Another important parameter that can be obtained from our results and that will be further used in the calibration of the simulations, is the estimated optimal volatility of the BCB. The range of estimated upper and lower daily thresholds suggest a 2.3% daily average change allowed by the BCB. That is, when the exchange rate change is equal to or above this value, the BCB will assume foreign exchange portfolio positions, signalling through order flow that it is acting to decrease volatility. Table 5 present descriptive statistics for our proxy of σ^2, that is $\hat{\sigma}^2$.

Table 5: Estimated Optimal Volatility

	$\hat{\sigma}^2$
Minimum	0,7%
P(25)	1,74%
Median	2,19%
Average	2,29%
P(75)	2,72%
Maximum	5,75%

Notes: Intra-daily BRL/USD rate recorded every 5 minutes.

The next section will explore, through model simulations, the decision-making process of the BCB. We will calibrate the model parameters using some of our own estimates as well as values taken from the respective literature. Simulations are provided as an alternative to the algebraic analytical evaluation of this non-linear problem. They will allow us to assess whether the observed accumulation of reserves by the BCB in the period between 2009 to 2011 is in line with the objective of controlling excess exchange rate volatility. We will also discuss, from these empirical findings, the accumulation of foreign swaps between 2013 and 2015. Although our simulations are performed for the BCB case, it is important to re-emphasise that our simple model can be applied (i.e., re-calibrated and/or extended) to a large range of CBs which have a similar *de jure* exchange rate regime[24].

5 Simulations

The results of the simulations below are shown for comparison with some statistics for the period that goes from Jan 2009 to July 2011. This period is characterised by a marked increase in commodity prices, favourable terms of trade and large surpluses in the Brazil's trade balance. IPEA (the Institute for Applied Economic Research of the Brazilian government) index of commodity prices, which excludes oil, rose from 166.61 in Jan 2009 to 242.57 in July 2011. In the same period, the Brazilian terms of trade, measured as the ratio of export to import prices, increased from 97.05 to 130.3, according to FUNCEX[25]. These facts help to explain the accumulated surplus in the trade balance during the aforementioned period, which reached 58.2 billion USD (495.14 billion USD of exports, according to BCB data). Trade surpluses were high during the adjoining months: 23.8 billion USD in 2008 and 24.6 billion USD accumulated in the twelve months after July 2011, meaning that customers could be bringing in larger proceeds of future exports contracts than those suggested by the trade balance only. From the 2nd of January 2009 until the 28th of July 2011, the BRL appreciated approximately 33% with respect to the USD.

[24] **R** codes are available with the authors upon request.

[25] A foundation which has an outstanding reputation in Brazil for the quality of its international trade studies.

Ex post and in sample it is almost certainly possible to fit optimal non-zero drifts in the exchange rates which will ameliorate forecasts with respect to the driftless random walk. One can also find evidence that out of sample forecasts with drifts will perform better than a naive driftless process. We have performed various forecasting exercises using mean absolute errors and mean squared errors[26]. The results are always sensitive to the starting point and sample split, however, the conclusions put forward above are generally valid. It must be stressed that the observed exchange rate is already a variable which has been subject to the intervention of the BCB. Hence, if the BCB applied a non-neutral policy function to some extent, at least part of the drift in the the exchange rate will "disappear" during the period, having been "transferred" to the foreign reserves.

We proceed with the simulations as if the BCB was perfectly informed about the drift and applied a non-neutral intervention function. For practical purposes, this is the same as if the BCB had no information whatsoever about the existence of a drift and, hence, it could not apply a non-neutral policy. Our choice is based on the fact that we are mainly interested in studying the impact of our intervention function during the period in which Brazil faced persistent and favourable commodity prices and, concomitantly, built up a large stock of foreign reserves. Clearly, as has been mentioned before, our model could be applied to other interesting BCB cases, such as during the most recent presidential impeachment case in 2016 (when a Peso problem was almost certainly in place).

Our simulation exercises will consider that γ_t is constant and that the BCB will react according to equations (11) and (12). The results for the case in which the BCB adopts a neutral policy when γ_t is imperfectly known will not be shown for the sake of space. However, inference on the likely consequences of a policy rule like (16) and (17) is straightforward. One part of γ_t would be absorbed in the exchange rate and the other on the level of foreign assets. As previously discussed, the latter impact depends on the information content of the order flow signals with respect to the drift.

The case of no drifts

We start by explaining the calibration of a "reference case" as presented in Table 6. The choice of $\gamma_t = 0, \forall t$ is analogous to the hypothesis of no long run trend in the level of the nominal exchange rate, i.e. a random walk without drift. The exchange rate change however, presents conditional time-dependent variance and unconditional constant variance. We used estimates provided by Ventura & Garcia (2012) for the coefficient that relates exchange rate changes to market order flow, (β) and adopted the same value for the intervention coefficient of the BCB (θ). The estimated SETAR model provides proxies for the thresholds that were initially used to calibrate σ^2: we used the average of the two estimated thresholds reported in Table 5 as a proxy for $\hat{\sigma}^2$. For the calibration of the GARCH parameters, we chose values for λ, α_0 and α_1 that were able to generate a proportion of around 50% of the intervention days out of the 1.000 in the sample - a similar proportion of intervention days has been found in Kohlscheen (2012). We report results only for the ARCH(1) model, which assumes, $\lambda = 0$,

[26] They are not reported in the paper but can be obtained with the authors upon request.

$\alpha_0 = 6$ and $\alpha_1 = 0,5$[27]. These parameters imply that the market order flow has zero mean and an unconditional standard deviation of 3,46 billion USD $\left(\sqrt{\frac{\alpha_0}{1-\alpha_1}} = \sqrt{\frac{6}{1-0,5}} \approx 3,46\right)$.

Table 6: Reference Case

Parameter	Value
γ, λ	0,00
α_0	6,00
α_1	0,50
$\hat{\sigma}^2$	0,02
β	1
θ	1

Notes: the value 1 for both parameters β and θ as seen above means that the 1 billion USD order flow impact on the exchange rate is 1%. ARCH structure was chosen to generate 53% of days with BCB intervention. The value of $\hat{\sigma}^2$ chosen according to our SETAR estimates in the previous section.

The main steps that we employed in our simulations can be summarised as following:

a. We initially created 1.000 order flow observations (η_t), calibrating both GARCH(1,1) and ARCH(1) parameters[28].

b. Using the order flow impact parameter (β), we simulated both the exchange rate variation and the realised volatility that would have prevailed at the end of each period in the absence of the BCB intervention.

c. We then evaluated the policy response function according to the cases shown in equations (11) and (12).

d. The BCB intervention order flow was aggregated over the 1.000 days in order to calculate the impact on the stock of foreign reserves. We also compared currency returns with and without intervention.

e. Steps (1) to (4) were repeated 20.000 times in order to obtain a measure of the distributions of the foreign reserve impact, the currency returns effect and the direction of interventions.

The results for the reference case are summarised in Figure 2. As can be seen, when there are only stochastic trends, i.e. there is no long-run deterministic trend in the nominal exchange rate, the accumulation of foreign reserves is, on average, close to zero. One can also see in Figure 2b that there is on average no significant predominance in either direction, when the exchange rate follows a driftless random walk. Finally, it can be seen that the CB

[27] Results with different specifications for the GARCH(1,1) model can be obtained with the authors upon request. We decided not to report these results as they normally differ only in terms of magnitude - richer dynamics tend to magnify the impact of interventions, by increasing volatility.

[28] There were 973 business days in Brazil during the period of the significant increase in reserves that is our primary concern in these simulations.

intervention lightens the weight of the tails of the currency returns distribution, with respect to the outcome that would prevail in the absence of a policy reaction function.

As mentioned, we understand that the magnitude of the signalling channel is given by the size of the θ parameter. We will analyse alternative scenarios for θ, $\theta < \beta$ and $\theta > \beta$, keeping constant the other parameters of the reference case. The next simulation, however, assumes $\theta = \beta$ and a deterministic drift ($\gamma_t = \gamma, \forall t$ and $\gamma \neq 0$).

Case 1. $\theta = \beta$ **and** $\gamma < 0$**:** The hypothesis $\theta = \beta$ implies that the information content of the CB order flow is the same as the market's. The CB's order flow will be of the same magnitude but with an opposite sign, neutralising the market order flow and leaving an exchange rate change, in absolute terms, equal to $|\sigma|$.

When $\gamma < 0$[29], the currency presents a long run appreciation trend. It is thus reasonable to expect that the CB would intervene predominantly in the positive (buying) direction, since the probability that the exchange rate will exceed the optimal volatility through the lower limit would be larger than through the upper threshold.

If the frequency of intervention is higher in one particular direction, we would observe an accumulation or depletion of foreign reserves along time depending on the sign and magnitude of the long-run deterministic trend. In the case of an appreciation trend, the CB would initiate buy trades more often than sell ones, thus accumulating stocks. This case is illustrated by Figure 3 where $\gamma = -0{,}0005$. In other words, even if η_t is zero on a particular day, the currency would still appreciate by 0,05% on that day. Over the 1.000 days, one would expect a cumulative appreciation of 50%.

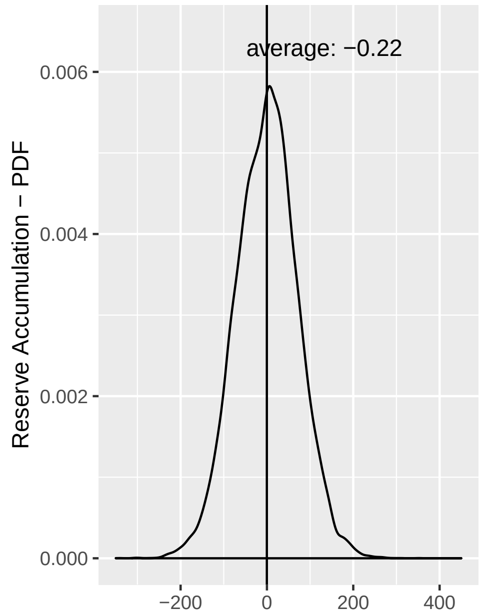
As can be observed, the average number of buys is greater than in the base scenario as well as the average accumulation, which is around 25.5 billion USD during the period. Only in a few out of the 20.000 simulations do the CB accumulated reserves that are over and above 150 billion USD. With regards to the shape of the currency returns distribution, one can see that the impact is qualitatively similar to the previous one. It is important to stress that, if the CB had not intervened, the average simulated appreciation in this period would have been 50,2%, a value close to the theoretical expected value, while the simulated appreciation with the CB intervention is 24,7%.

Case 2. $\theta < \beta$ **and** $\gamma < 0$**:** The assumption $\theta < \beta$ implies that the CB has some limitation in conveying signals to the market. The monetary authority would have to generate a larger order flow than in the $\theta = \beta$ case, in order to affect the currency in the same magnitude. Since the CB needs to generate a larger intervention flow, the accumulation of foreign reserves will be faster than before. This case is summarised in Figure 4 which assumes $\gamma = -0{,}0005$ and a coefficient equivalent to 50% of the market's order flow.

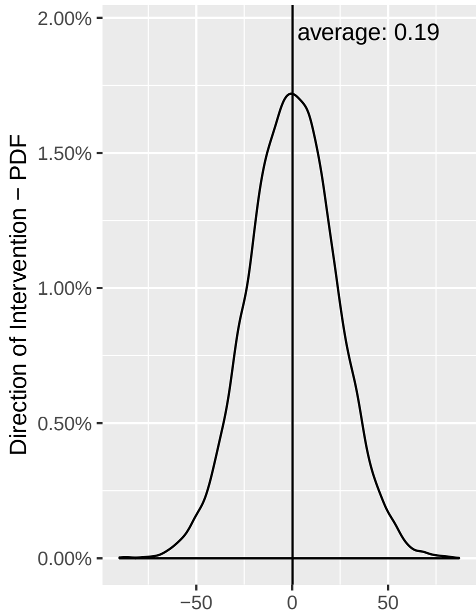
Figure 4 shows that the average accumulation is around 52,1 billion USD. Most of the interventions occur in the buying direction. The less credible intervention affects currency

[29] We only present results for negative values of γ, since the results obtained for the opposite sign and same magnitude, *ceteris paribus*, are symmetrical.

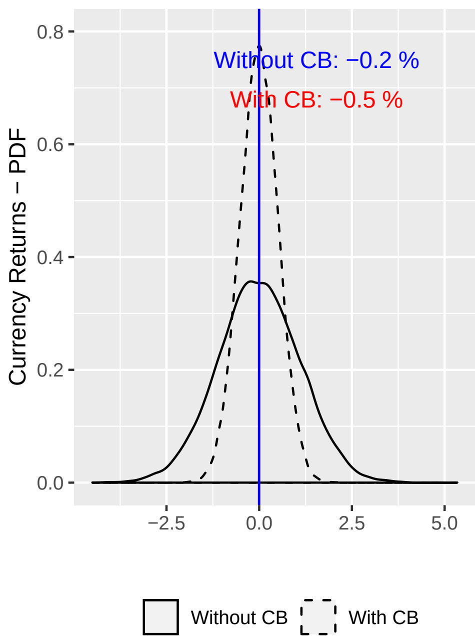
Figure 2: Simulation results for $\theta = \beta = 1$ and $\gamma = 0$



(a) accumulation of foreign reserves



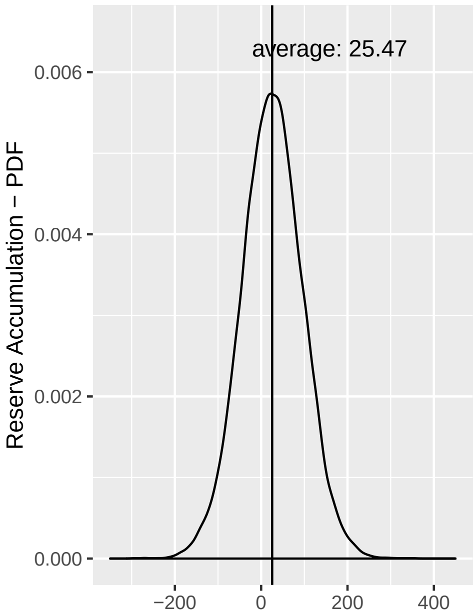
(b) direction of the interventions



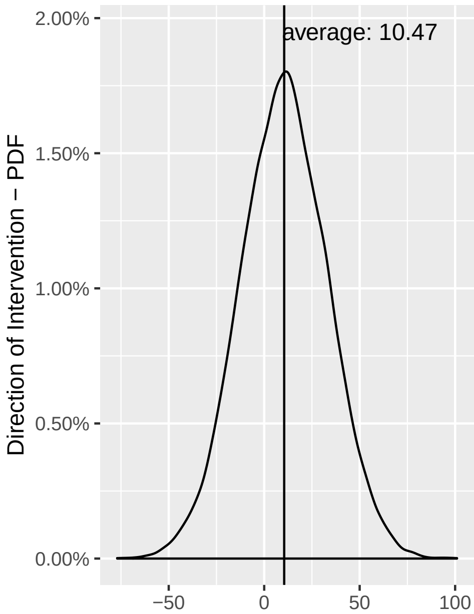
(c) currency returns

Notes: Figure 2a presents the distribution of the accumulated reserves by the CB. Figure 2b shows the distribution of the direction of the interventions for each of the 20.000 "experiments" of the 1.000 days. This variable was constructed by assuming the value (+1) for the CB initiated-buy trades of foreign currency, (-1) for the CB initiated-sells and (0) in the case of no intervention. Figure 2c presents a comparison between the currency returns, in percentage changes, with and without intervention from the CB.

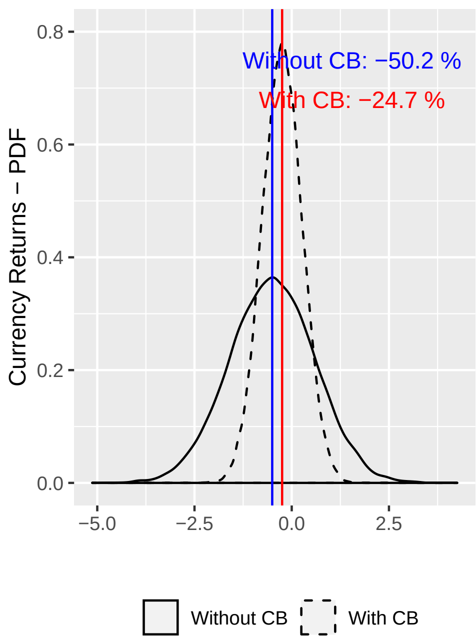
Figure 3: Simulation results for $\theta = \beta = 1$ and $\gamma = -0,0005$



(a) accumulation of foreign reserves



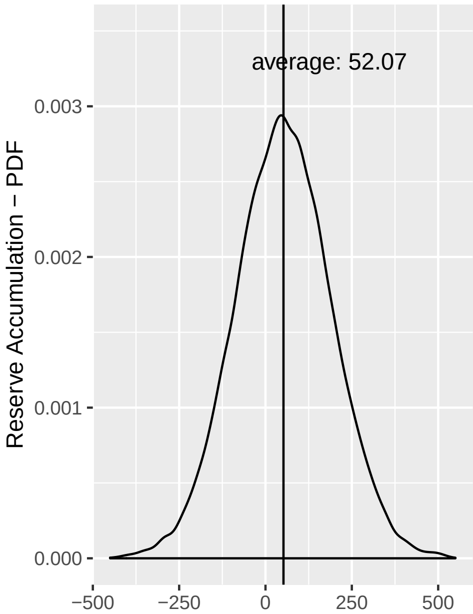
(b) direction of the interventions



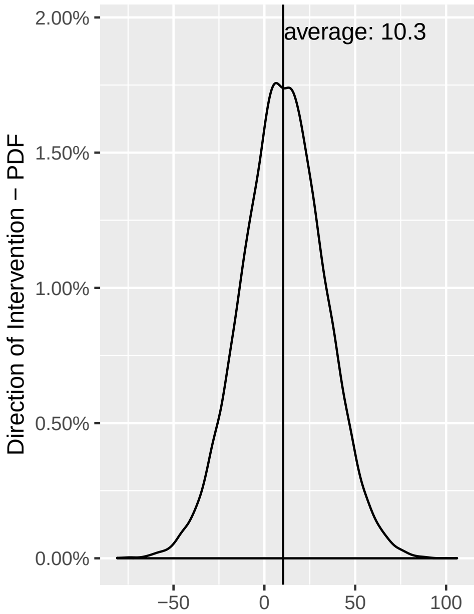
(c) currency returns

Observation: *see notes in Figure 2.*

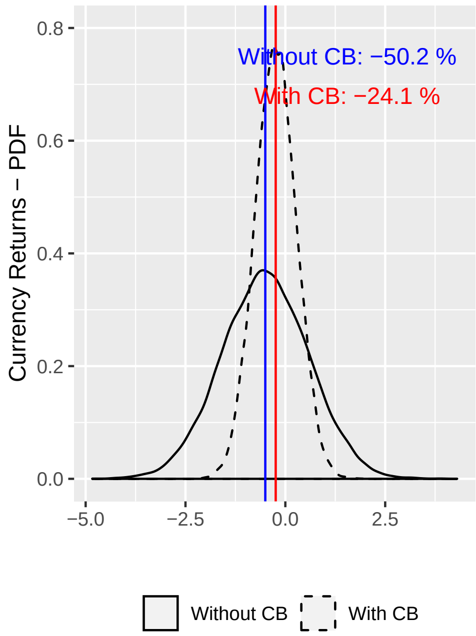
Figure 4: Simulations Results for $\theta = \frac{1}{2}\beta = 0.5$ and $\gamma = -0,0005$



(a) accumulation of foreign reserves



(b) direction of the interventions



(c) currency returns

Observation*: see notes in Figure 2.*

returns in the same qualitative way as before, with an average appreciation with the CB of 24,1% and 50,2% without it.

Case 3. $\theta > \beta$ e $\gamma < 0$: This scenario considers that the information content of the BCB order flow is larger than that of the market. This case is based on the fact that the BCB is the largest player and probably has access to a larger information set than any other single agent in the market[30]. Another interpretation is that the BCB is perceived as having a high degree of credibility in delivering the desired volatility control. That is, $\theta > \beta$ implies that the BCB is efficient in signalling the correct direction and magnitude of its interventionist policy. Considering $\gamma = -0,0005$, $\theta = 1.25$ and $\beta = 1$, such that the impact of the market order flow is only 80% of that of the BCB, gives the results presented in Figure 5.

As expected, the average accumulation is lower than in the previous case, reaching 20,74 billion USD in this simulation. Other results are similar to the previous two cases. We stress that without the BCB, the average appreciation is 50,3%, while the average negative change in the nominal exchange rate with the intervention is 24,4%. That is, the collateral effect of the BCB's interventions is to decrease average currency returns.

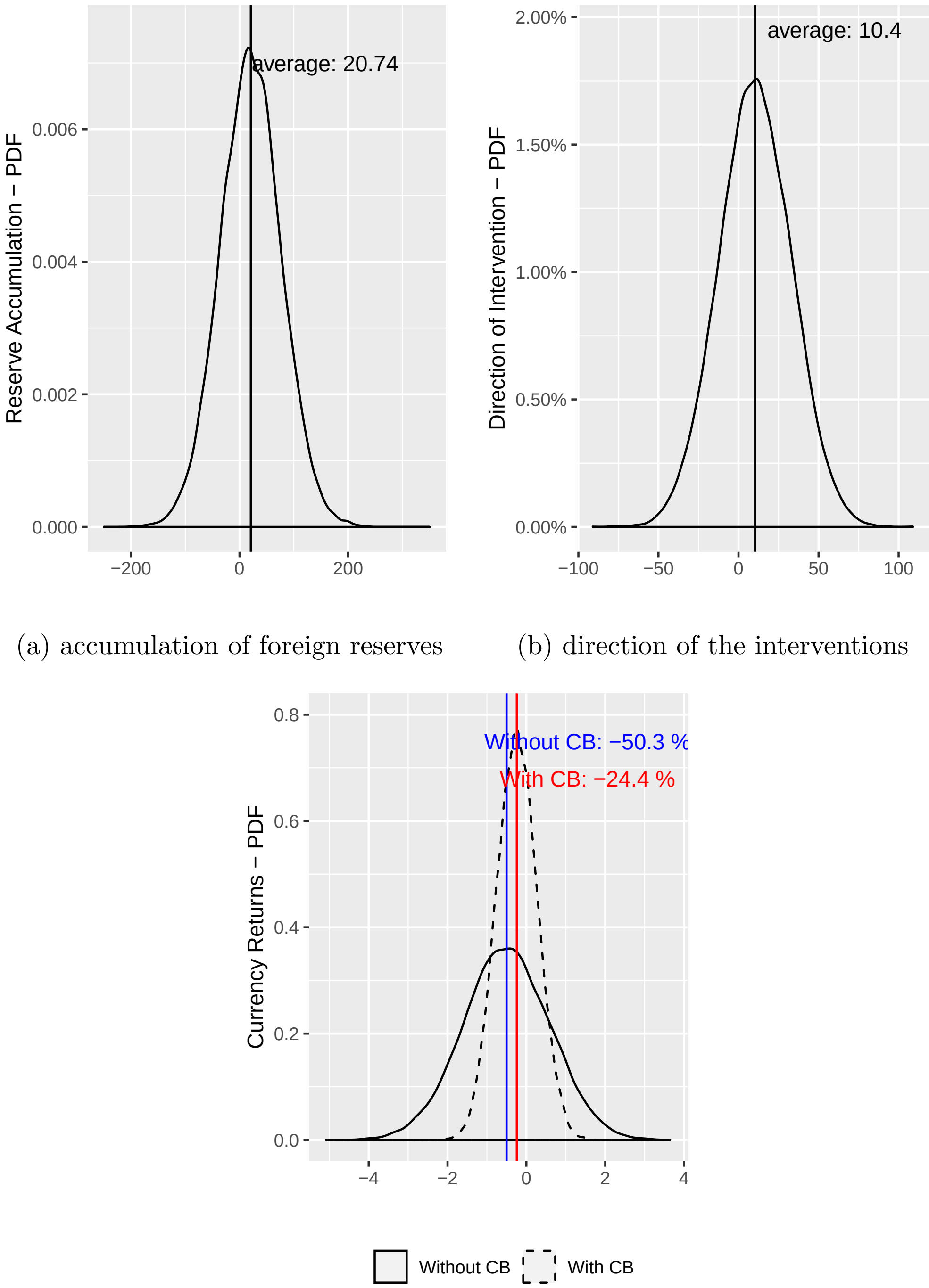
As previously stated, between January 2009 and July 2011, the BCB reserves increased by approximately 150 billion USD while the currency appreciated about 22%. Each line of Table 7 presents selected descriptive statistics for each of the (20.000×1.000) simulations using different values for γ, β and θ. Careful analysis of Table 7 shows that the larger the long run trend is, the higher the stock of foreign reserves will become. It is interesting to note that the probability of the BCB accumulating 150 USD billions or more of foreign reserves is high (being larger than 50% in some cases) when the signalling channel is weak. One can see that our simulation results in Table 7 can successfully explain the average direction of the changes in the observed data: exchange rates and level of foreign reserves. Actually, it is not difficult to replicate the observed amount of currency appreciation. However, we are not able to replicate, on average, the substantial change in the level of foreign assets held by the BCB, only by chance.

There are many possible explanations for these findings that are complementary to the ones put forward in the Introduction. One is that, during the period of analysis, the market already expected some daily intervention from the BCB which significantly weakened the signalling channel. This could be due to the knowledge of a small negative deterministic daily time trend γ. As a matter of fact, this was an outstanding period for commodity exporters like Brazil, who experienced improved terms of trade and fast growth of the *quantum* exported. One hypothesis is that it would have been harder for the BCB to provide an effective signal through initiated-buys, in other words, θ would be a negative function of the absolute value of γ.

Another plausible explanation is that the BCB actually had another objective, different from the one stated in the optimization problem. This could be, for example, the goal of accumulating foreign reserves (for precautionary reasons, as previously stated) by deliberately

[30] Specially because **all** customer orders in the primary market, which will later be fed into interdealer order flow, must be recorded in a BCB electronic system.

Figure 5: Simulations Results for $\theta = 1.25, \beta = 1$ and $\gamma = -0,0005$



(a) accumulation of foreign reserves

(b) direction of the interventions

(c) currency returns

Observation: *see notes in Figure 2.*

offsetting the aforementioned appreciating deterministic trend. In fact, there are many other explanations that could be given and which could be linked to the parameter values chosen for the calibration of the simulations. For example, a higher appreciation trend would explain larger interventions and faster reserve accumulation, *ceteris paribus.*

The period known as the "daily ration", which goes from August 2013 until April 2015, registered a total of 115 billion USD in currency swaps[31]. During this period, the BRL depreciated by approximately 35%, which, according to inference based on our model and the results obtained in the previous simulations, suggests that the BCB would have intervened by generating significant sell flows. As the period was characterised by a massive increase in the number of currency swaps, there was no marked change in the level of foreign reserves. Although our simulations correctly predict the average direction of the interventions, we were also not able to replicate - on average - the massive increase in the number of swap contracts, at least using parameters that seem to be in accordance to the available evidence[32]. The justification is analogous to the ones put forward above. It must be also stressed that the country went through a period of political and economic uncertainty which could have affected the results in many different ways: for example by reducing the impact of the signalling channel or leading to large outliers.

[31] According to information provided by the president of the BCB in a statement to the Federal Senate in 24^{th} of march 2015.

[32] For these simulations, we considered that $I_t^{(j)}$ was divided into two components: z_t and w_t, representing the spot intervention and the net result of traditional and reverse swap operations at date t, respectively. We also assumed that market interventions through traditional currency swap operations had an analogous impact to spot market interventions, the main difference being the absence of changes in the stock of USD and BRL currencies. Hence, unless one of the parties need currency for some other non-portfolio balancing reason, agents should be indifferent between currency swaps and spot operations. We implicitly considered that the CB first decided on the composition of the intervention, i.e. via currency swap or through the spot market, and then its magnitude. Although we do not propose a mechanism as to which composition to choose, one simple way that we used to incorporate these two instruments was to define that $I_t^{(j)} \equiv \psi_1 z_t + \psi_2 w_t$, where ψ_1 and ψ_2 are parameters that capture the extent to which the magnitude of each type of intervention affects exchange rates.

Table 7: Summary of the Simulation Results - ARCH(1)

γ	β	θ	Average Reserve Accumulation - billions of USD (b)	Direction of Intervention	Average with BCB (%)	Average without BCB (%)	$P(b) >$ 150 USD billions (%)
0	1	1	0.22	0.60	-0.27	-0.52	1.76
0	1	0.6	-1.52	-0.13	0.19	1.10	9.85
0	1	0.2	-2.81	-0.01	-0.09	0.47	33.11
0	0.8	0.8	-0.38	-0.22	0.62	0.93	0.88
0	0.8	0.48	-0.64	-0.02	0.09	0.40	7.47
0	0.8	0.16	4.00	0.25	-0.36	-1.00	32.13
0	0.6	0.6	-0.47	-0.15	0.51	0.80	0.20
0	0.6	0.36	-0.06	-0.04	0.07	0.09	4.21
0	0.6	0.12	-2.98	-0.29	0.74	1.10	27.96
0	0.4	0.4	-0.11	0.00	-0.08	-0.04	0.00
0	0.4	0.24	-0.25	0.11	-0.32	-0.26	0.77
0	0.4	0.08	0.27	0.03	-0.13	-0.16	21.02
-0.05	1	1	25.97	10.35	-24.50	-50.50	3.64
-0.05	1	0.6	44.56	10.74	-24.88	-51.62	18.29
-0.05	1	0.2	129.91	10.50	-24.70	-50.68	47.74
-0.05	0.8	0.8	26.05	11.29	-29.06	-49.90	2.40
-0.05	0.8	0.48	43.79	11.29	-29.25	-50.28	15.35
-0.05	0.8	0.16	129.77	11.28	-29.40	-50.17	47.42
-0.05	0.6	0.6	22.92	11.14	-35.59	-49.34	0.74
-0.05	0.6	0.36	40.40	11.56	-36.42	-50.96	10.20
-0.05	0.6	0.12	119.80	11.47	-36.30	-50.68	45.36
-0.05	0.4	0.4	15.11	7.95	-43.83	-49.87	0.01
-0.05	0.4	0.24	25.71	7.98	-43.72	-49.89	2.23
-0.05	0.4	0.08	76.60	8.12	-44.07	-50.19	34.58
-0.1	1	1	51.27	20.79	-48.70	-99.97	8.12
-0.1	1	0.6	85.74	20.51	-48.21	-99.65	29.19
-0.1	1	0.2	255.77	20.74	-49.05	-100.20	61.80
-0.1	0.8	0.8	52.92	22.69	-58.67	-101.00	6.25
-0.1	0.8	0.48	85.56	22.22	-57.46	-98.53	26.72
-0.1	0.8	0.16	258.96	22.53	-58.11	-99.54	63.52
-0.1	0.6	0.6	47.51	22.71	-71.83	-100.34	2.44
-0.1	0.6	0.36	80.01	22.69	-71.52	-100.32	21.16
-0.1	0.6	0.12	240.31	22.76	-71.57	-100.41	63.33
-0.1	0.4	0.4	30.93	16.20	-87.65	-100.02	0.07
-0.1	0.4	0.24	51.62	16.09	-87.78	-100.17	5.38
-0.1	0.4	0.08	154.98	16.05	-87.86	-100.26	51.07

Notes: 20.000 simulations for 1.000 days. γ is the daily % constant change; β and θ refer to the impact of a one billion USD order flow on the exchange rate ($\beta = \theta = 1$, for example, implies that a 1 billion USD order flow changes the exchange rate by 1%); mean with and without CB are the averages for a total of 1.000 days in percentage terms; the probability of an accumulation of 150 USD billions in foreign reserves in the last column is also expressed in percentages.

6 Concluding Remarks

We presented a simple but innovative and, hopefully, practical model that can help CBs across the world to understand the stock and flow implications of a "flexible" exchange rate regime that adopts a smoothing volatility policy. Our results imply that the exchange rate follows a non-linear dynamics and that small daily deterministic trends in the exchange rate can have a large impact on the stock of foreign assets in a matter of a few months. Simulations show that either a non-neutral or an unaware (with respect to the drift) interventionist smoothing policy affect both moments of the exchange rate change (or the currency returns distribution). These effects can also be large according to our findings.

We were also able to show that, on average, CB interventionist policies could only have affected volatility, without changing the level of the exchange rate and the amount of foreign reserves, if there were no deterministic trends in the data generating process of the exchange rate. During the period of large swings in the stocks of foreign assets in emerging economies, and particularly for the case of Brazil, there are some possible explanations: 1) small but persistent daily appreciating trends during the accumulation period, probably due to the observed large swing in commodity prices, and a depreciating one during the de-accumulation episode studied in this paper, the latter probably due to a change in the level of FX hedge demand; 2) the BCB effectively partially offset the impact of these trends on the BRL for other reasons than only being unaware of the drift, such as the intentional build up of precautionary savings; 3) the signalling channel of the BCB was small. These explanations are non-exhaustive and non-mutually exclusive.

A future direction of investigation should consider extending the model to incorporate a richer description of the FX market. This could include the relationship of the spot rate with other variables such as the forward rate, which represents a large and liquid market in many markets. One could develop more interesting and complex learning processes for the CB with regards to the drift term. It would be interesting to investigate whether and how the drifts are related to permanent interest rate differentials in small open economies. Peso problems and speculative attacks also seem to be promising modelling options for the drifts. Finally, one could integrate other social welfare weights in the social welfare function of the monetary authority, given capital mobility.

References

Aizenman, J. (2011), The Impossible Trinity- from the Policy Trilemma to the Policy Quadrilemma, Technical report, Department of Economics, UC Santa Cruz.
URL: *https://ideas.repec.org/p/cdl/ucscec/qt8cq7g4c9.html*

Alfaro, L. & Kanczuk, F. (2018), 'Debt redemption and reserve accumulation', *IMF Economic Review (forthcoming)* (19098).
URL: *https://EconPapers.repec.org/RePEc:nbr:nberwo:19098*

Amador, M., Bianchi, J., Bocola, L. & Perri, F. (2016), 'Reverse speculative attacks', *Journal of Economic Dynamics and Control* **72**, 125 – 137.
URL: *http://www.sciencedirect.com/science/article/pii/S0165188916300525*

Andersen, T. G., Bollerslev, T., Diebold, F. X. & Labys, P. (2003), 'Modeling and Forecasting Realized Volatility', *Econometrica* **71**(2), 579–625.
URL: *https://ideas.repec.org/a/ecm/emetrp/v71y2003i2p579-625.html*

Bacchetta, P. & van Wincoop, E. (2006), 'Can information heterogeneity explain the exchange rate determination puzzle?', *American Economic Review* **96**(3), 552–576.
URL: *https://EconPapers.repec.org/RePEc:aea:aecrev:v:96:y:2006:i:3:p:552-576*

Baillie, R. & Osterberg, W. P. (1997), 'Why do central banks intervene?', *Journal of International Money and Finance* **16**(6), 909–919.
URL: *https://EconPapers.repec.org/RePEc:eee:jimfin:v:16:y:1997:i:6:p:909-919*

Barroso, J. (2014), Realized volatility as an instrument to official intervention, Working Papers Series 363, Central Bank of Brazil, Research Department.
URL: *https://EconPapers.repec.org/RePEc:bcb:wpaper:363*

Berganza, J. C. & Broto, C. (2012), 'Flexible inflation targets, forex interventions and exchange rate volatility in emerging countries', *Journal of International Money and Finance* **31**(2), 428 – 444.
URL: *http://www.sciencedirect.com/science/article/pii/S0261560611001860*

Calvo, G. A. & Reinhart, C. M. (2002), 'Fear of floating', *The Quarterly Journal of Economics* **117**(2), 379–408.

Calvo, G., Izquierdo, A. & Loo-Kung, R. (2013), 'Optimal holdings of international reserves: Self-insurance against sudden stops', *Monetaria* **I**(1), 1–35.
URL: *https://EconPapers.repec.org/RePEc:cml:moneta:v:i:y:2013:i:1:p:1-35*

Chamon, M., Garcia, M. & Souza, L. (2017), 'FX interventions in Brazil: a synthetic control approach', *Journal of International Economics* **108**(C), 157–168.
URL: *https://EconPapers.repec.org/RePEc:eee:inecon:v:108:y:2017:i:c:p:157-168*

Daude, C., Yeyati, E. L. & Nagengast, A. J. (2016), 'On the effectiveness of exchange rate interventions in emerging markets', *Journal of International Money and Finance* **64**, 239 – 261.
URL: *http://www.sciencedirect.com/science/article/pii/S0261560616000139*

Dominguez, K. & Frankel, J. (1993), 'Does foreign-exchange intervention matter? the portfolio effect', *American Economic Review* **83**(5), 1356–69.
URL: *https://EconPapers.repec.org/RePEc:aea:aecrev:v:83:y:1993:i:5:p:1356-69*

Dominguez, K. M. (1990), 'Market responses to coordinated central bank intervention', *Carnegie-Rochester Conference Series on Public Policy* **32**, 121 – 163.
URL: *http://www.sciencedirect.com/science/article/pii/016722319090024F*

Dominguez, K. M. (2006), 'When do central bank interventions influence intra-daily and longer-term exchange rate movements?', *Journal of International Money and Finance* **25**(7), 1051 – 1071.
URL: *http://www.sciencedirect.com/science/article/pii/S0261560606000738*

Duarte, P. (2009), 'A feasible and objective concept of optimal monetary policy: the quadratic loss function in the postwar period', *History of Political Economy* **41**(1), 1–55.
URL: *https://EconPapers.repec.org/RePEc:hop:hopeec:v:41:y:2009:i:1:p:1-55*

Evans, M. D. D. (2011), *Exchange Rate Dynamics*, number 9475 *in* 'Economics Books', Princeton University Press.
URL: *https://ideas.repec.org/b/pup/pbooks/9475.html*

Evans, M. D. D. & Lyons, R. K. (2002), 'Order flow and exchange rate dynamics', *Journal of Political Economy* **110**(1), 170–180.
URL: *http://ideas.repec.org/a/ucp/jpolec/v110y2002i1p170-180.html*

Fratzscher, M., Gloede, O., Menkhoff, L., Sarno, L. & Stöhr, T. (2019), 'When is foreign exchange intervention effective? Evidence from 33 countries', *American Economic Journal: Macroeconomics, fothcoming* .

Glick, R. & Hutchison, M. (1989), 'Exchange rates and monetary policy', *Economic Review* (Spr), 17–29.
URL: *https://ideas.repec.org/a/fip/fedfer/y1989isprp17-29.html*

Hansen, B. (1999), 'Testing for linearity', *Journal of Economic Surveys* **13**(5), 551–576.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-6419.00098*

Hung, J. H. (1997), 'Intervention strategies and exchange rate volatility: a noise trading perspective', *Journal of International Money and Finance* **16**(5), 779 – 793.
URL: *http://www.sciencedirect.com/science/article/pii/S0261560697000235*

Hur, S. & Kondo, I. O. (2016), 'A theory of rollover risk, sudden stops, and foreign reserves', *Journal of International Economics* **103**, 44 – 63.
URL: *http://www.sciencedirect.com/science/article/pii/S0022199616301040*

IMF (2017), 'Annual report on exchange arrangements and exchange rate restrictions', *International Monetary Fund* .

Jeanne, O. & Rancière, R. (2011), 'The optimal level of international reserves for emerging market countries: a new formula and some applications', *Economic Journal* **121**(555), 905–930.
URL: *https://ideas.repec.org/a/ecj/econjl/v121y2011i555p905-930.html*

Kearns, J. & Rigobon, R. (2005), 'Identifying the efficacy of central bank interventions: evidence from australia and japan', *Journal of International Economics* **66**(1), 31 – 48.
URL: *http://www.sciencedirect.com/science/article/pii/S0022199604000777*

Klein, M. W. & Shambaugh, J. C. (2015), 'Rounding the corners of the policy trilemma: sources of monetary policy autonomy', *American Economic Journal: Macroeconomics* **7**(4), 33–66.
URL: *http://www.aeaweb.org/articles?id=10.1257/mac.20130237*

Kohlscheen, E. (2012), Order flow and the Real: indirect evidence of the effectiveness of sterilized interventions, Working Papers Series 273, Central Bank of Brazil, Research Department.
URL: *https://EconPapers.repec.org/RePEc:bcb:wpaper:273*

Kohlscheen, E. & Andrade, S. C. (2014), 'Official FX interventions through derivatives', *Journal of International Money and Finance* **47**(C), 202–216.
URL: *https://EconPapers.repec.org/RePEc:eee:jimfin:v:47:y:2014:i:c:p:202-216*

Korinek, A. & Servén, L. (2016), 'Undervaluation through foreign reserve accumulation: static losses, dynamic gains,', *Journal of International Money and Finance* **64**, 104 – 136.
URL: *http://www.sciencedirect.com/science/article/pii/S026156061600036X*

Krugman, P. (1996), Are currency crises self-fulfilling?, *in* 'NBER Macroeconomics Annual 1996, Volume 11', NBER Chapters, National Bureau of Economic Research, Inc, pp. 345–407.
URL: *https://ideas.repec.org/h/nbr/nberch/11032.html*

Kydland, F. & Prescott, E. (1977), 'Rules rather than discretion: the inconsistency of optimal plans', *Journal of Political Economy* **85**(3), 473–91.
URL: *https://EconPapers.repec.org/RePEc:ucp:jpolec:v:85:y:1977:i:3:p:473-91*

Levy-Yeyati, E., Sturzenegger, F. & Gluzmann, P. A. (2013), 'Fear of appreciation', *Journal of Development Economics* **101**, 233 – 247.
URL: *http://www.sciencedirect.com/science/article/pii/S0304387812001022*

Meese, R. A. & Rogoff, K. (1983), 'Empirical exchange rate models of the seventies: do they fit out of sample?', *Journal of International Economics* **14**(1), 3 – 24.
URL: *http://www.sciencedirect.com/science/article/pii/002219968390017X*

Menkhoff, L. (2013), 'Foreign exchange intervention in emerging markets: a survey of empirical studies', *The World Economy* **36**(9), 1187–1208.
URL: *https://EconPapers.repec.org/RePEc:bla:worlde:v:36:y:2013:i:9:p:1187-1208*

Mussa, M. (1981), *The role of official intervention*, NY: Group of Thirty.

Obstfeld, M. & Rogoff, K. (2000), The six major puzzles in international macroeconomics: is there a common cause?, NBER Working Papers 7777, National Bureau of Economic Research, Inc.
URL: *https://ideas.repec.org/p/nbr/nberwo/7777.html*

Obstfeld, M., Shambaugh, J. C. & Taylor, A. M. (2010), 'Financial stability, the Trilemma, and international reserves', *American Economic Journal: Macroeconomics* **2**(2), 57–94.
URL: *https://ideas.repec.org/a/aea/aejmac/v2y2010i2p57-94.html*

Oliveira, F. N. d. & Plaga, A. (2011), 'Eficácia das intervenções do Banco Central do Brasil sobre a volatilidade condicional da taxa de câmbio nominal', *Revista Brasileira de Economia* **65**(1), 71–92.

Reitz, S. & Taylor, M. P. (2008), 'The coordination channel of foreign exchange intervention: a nonlinear microstructural analysis', *European Economic Review* **52**(1), 55 – 76.
URL: *http://www.sciencedirect.com/science/article/pii/S0014292107001018*

Rey, H. (2015), Dilemma not Trilemma: the global financial cycle and monetary policy independence, Working Paper 21162, National Bureau of Economic Research.
URL: *http://www.nber.org/papers/w21162*

Sarno, L. & Taylor, M. (2001), 'Official intervention in the foreign exchange market: is it effective and, if so, how does it work?', *Journal of Economic Literature* **39**(3), 839–868.
URL: *https://EconPapers.repec.org/RePEc:aea:jeclit:v:39:y:2001:i:3:p:839-868*

Simonassi, A. G., Silva, J. H. F. & Arraes, R. A. (2012), 'Dinâmica da taxa de câmbio no Brasil sob o regime de câmbio flutuante', *Pesquisa e Planejamento Econômico* **42**(2).

Taylor, J. B. (1998), An historical analysis of monetary policy rules, NBER Working Papers 6768, National Bureau of Economic Research, Inc.
URL: *https://ideas.repec.org/p/nbr/nberwo/6768.html*

Tryon, R. W. (1983), Small empirical models of exchange market intervention: a review of the literature, Staff Studies 134, Board of Governors of the Federal Reserve System (U.S.).
URL: *https://EconPapers.repec.org/RePEc:fip:fedgss:134*

Ventura, A. & Garcia, M. (2012), 'Mercados futuro e à vista de câmbio no brasil: o rabo balança o cachorro', *Revista Brasileira de Economia* **66**(1), 21–48.
URL: *http://bibliotecadigital.fgv.br/ojs/index.php/rbe/article/view/1907*

Vitale, P. (2008), 'The impact of FX intervention on FX markets: a market microstructure analysis', *International Journal of Finance & Economics* **16**(1), 41–62.